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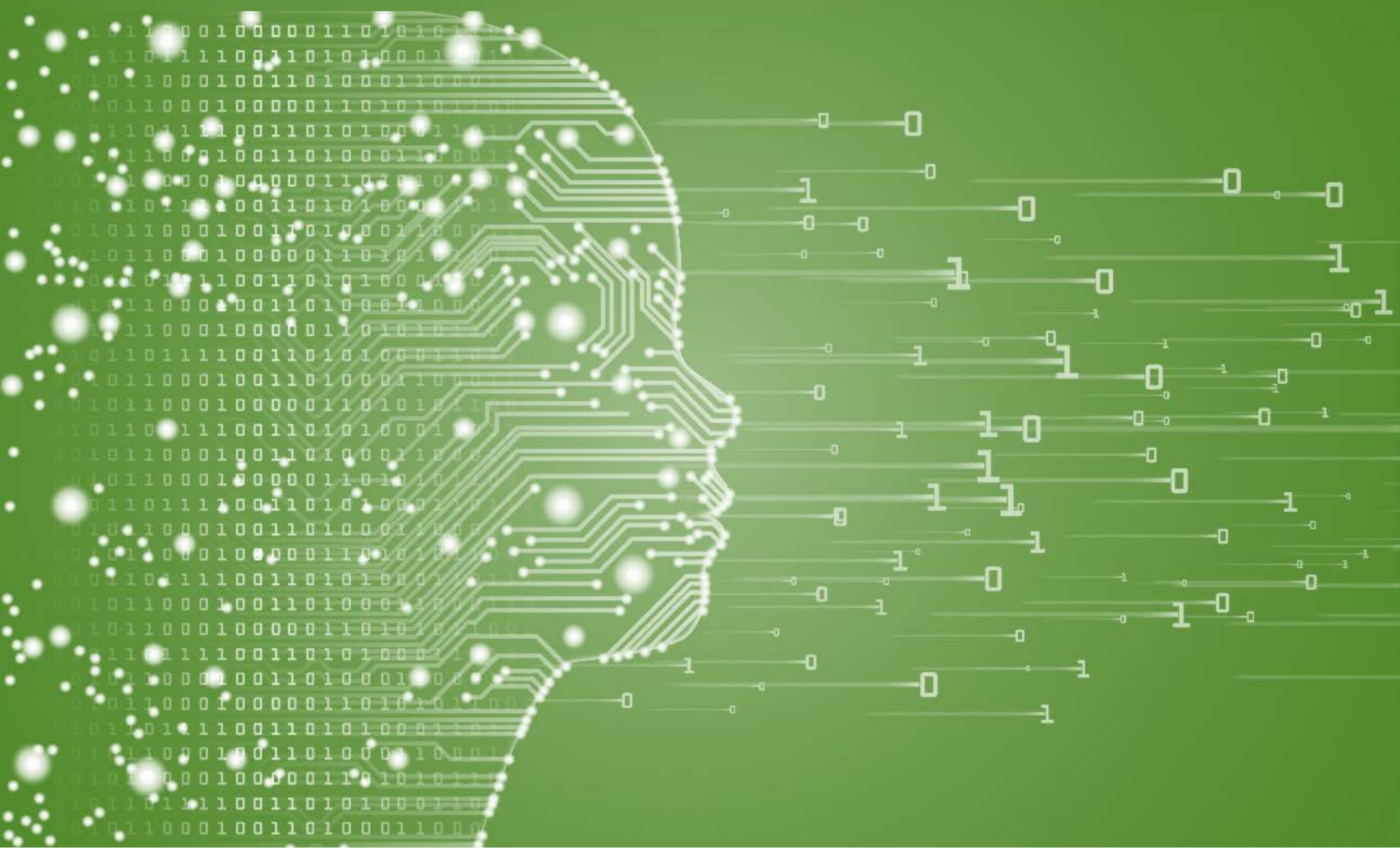

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CONTENTS

Original scientific papers

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Data mining approach in detecting inaccurate financial statements in government-owned enterprises

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Abstract. The study aims to assess the capability of various data mining techniques in detecting inaccurate financial statements of government-owned enterprises operating in the Federation of Bosnia and Herzegovina (FBiH). Inaccurate financial statements indicate potential financial fraud. Prediction models of four classification algorithms (J48, KNN, MLP, and BayesNet) were examined using a dataset comprising 200 audited financial statements from government-owned enterprises under the supervision of the Audit Office of the Institutions in the Federation of Bosnia and Herzegovina. The results obtained through data mining analysis reveal that a dataset encompassing seven balance sheet items provides the most comprehensive depiction of financial statement quality. These seven attributes are: opening entry of accounts receivable, profit (loss) at the end of the period, operating assets at the end of the period, accounts receivable at the end of the period, opening entry of operating assets, short term financial investments at the end of the period, and opening entry of short-term financial investments. By employing these seven attributes, the MLP algorithm was implemented to construct the most precise predictive model, achieving a 76% accurate classification rate for financial statements. Leveraging the identified attributes, a mathematical model could potentially be formulated to effectively predict financial statements of government-owned enterprises in FBiH. This, in turn, could considerably facilitate the process of selecting GOEs for inclusion in the annual work plan of state auditors. Presently, due to resource constraints, government-owned enterprises in FBiH do not undergo regular annual scrutiny by state auditors, with only 10 to 15 such enterprises being subject to audits each year. The results of this research can also be beneficial to both the public and the Financial Intelligence Agency in the FBiH. The paper contributes to filling the gap in the literature regarding the applied methodology, particularly in the part concerning the attributes used in the research.

Keywords: data mining, financial statement frauds, government-owned enterprises, prediction of financial statements accuracy

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Original scientific paper.

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1. Introduction

Transitional countries, including Bosnia and Herzegovina, often face problems such as an inefficient public sector, a high corruption index, and the employment of politically affiliated personnel in the management structure and supervisory boards of government-owned enterprises. The resources of these enterprises are often used to support political campaigns and favor certain suppliers in the procurement of goods and services, under the guise of the Public Procurement Law. The challenges faced by transitional countries like Bosnia and Herzegovina, including an inefficient public sector, high corruption index, and politically affiliated personnel in government-owned enterprises, have been widely documented [3]. These issues have eroded public trust in state institutions and led to financial embezzlement in government-owned enterprises. Anti-corruption policies and good governance practices have been implemented to address these challenges, but their effectiveness in rebuilding trust in public institutions remains a subject of debate [3]. The restructuring of state-owned enterprises, with a focus on corporate governance and the role of the state, has been proposed as a potential solution. However, the complex relationship between tax evasion, state capacity, and trust in transitional countries, as well as the prevalence of public procurement corruption in Bosnia and Herzegovina, further complicate the situation. Government-owned enterprises that are owned by the state or lower levels of government are frequently accused of financial embezzlement and do not enjoy public trust in the accuracy of their financial statements. Financial statement fraud involves the intentional concealment or omission of critical information resulting from a deliberate failure to report financial data in line with generally accepted accounting principles. Financial fraud is a serious problem worldwide and is particularly pronounced in companies that are state-owned in countries such as Bosnia and Herzegovina. Lalić, Jovičić & Bošnjaković [20] highlights the inflow of money from abroad and the presentation of operating losses as common examples of financial fraud in the region. Buljubašić Musanović & Halilbegović [2] further underscores the manipulation of financial statements in failing small and medium-sized enterprises, with significant differences in accruals, asset quality, leverage, profitability, and liquidity between failing and non-failing SMEs. Isaković-Kaplan et al. [13] explores the application of Benford's Law in detecting potential earnings manipulation in income statements of economic entities, emphasizing the need for additional forensic investigations. Yadiati, Rezwiandhari & Ramdany [35] highlight a broader perspective by identifying factors such as financial stability, external pressure, industry nature, director changes, and collaboration with government projects as possible indicators of fraudulent financial reporting in state-owned enterprises. Another issue also arises from the fact that financial statements of government-owned enterprises are not subject to annual audits by state auditors. Due to the extensive number of institutions under state audit supervision (more than 2,000 institutions), only a few of government-owned enterprises are incorporated into the annual audit plan. The Audit Office of the Institutions in the Federation of Bosnia and Herzegovina does not utilize modern data mining tools to aid in the detection of inaccurate financial statements. In the process of planning which government-owned enterprises will be included in the audit plan for the year, state auditors are guided by media reports, anonymous complaints, and internal information from previous financial audits. The fundamental research question in this study is which balance sheet items and data mining technique provide the best probability of predicting inaccurate financial statements in government-owned enterprises? Therefore, the objectives of this paper are to identify the balance sheet items that best indicate inaccurate financial statements and to determine which data mining technique achieves the best results in predicting inaccuracies in financial statements for government-owned enterprises. Data mining techniques will be employed to detect balance sheet items from financial position reports of enterprises that offer the most accurate predictions of the state auditors' assessments of financial statement quality. Furthermore, various data mining techniques will be utilized to evaluate their predictive efficacy through comparative analysis of forecasting results. This paper aims to contribute to addressing the literature gap concerning the attributes used to identify balance sheet items that best predict inaccuracies in financial statements of government-owned enterprises in FBiH. These objectives will be achieved through a systematic literature review, identifying gaps in the literature. Following this, a detailed explanation of the research methodology and the application of data mining techniques will follow. Finally, the research findings will be presented, accompanied by their explanations.

2. Literature review

A range of studies have explored the use of various models and techniques to predict inaccurate financial statements based on audit opinion. Gadžo et al. [5] found that the Beneish M-score model, particularly its partial indicators, can accurately predict the quality of financial statements in public enterprises. Similarly, Sánchez-Serrano et al. [27] developed a model for predicting audit opinion in consolidated financial statements, achieving high accuracy. Wu & Li [33] further improved on this by using a BP neural network with Adam optimizer to predict audit opinions in listed companies, with a high accuracy rate. Yue, Shen & Chu [37] also identified specific financial ratios, such as net assets per share and earnings per share, as strong indicators of false financial affairs, on the basis of the model of logistic regression analysis. Research consistently shows that financial ratios are a key factor in predicting an auditor's qualified opinion on financial statements [6]. These ratios, such as retained earnings to total assets, equity to total liabilities, and net income to total assets, are used in various models to accurately classify qualified and unqualified opinions. Rudkhani & Jabbari [6] found that only two financial ratios, "earnings per share" and "fixed asset turnover," were needed for an accuracy rate of 64.1% . Similarly, Gadžo [5] achieved a high accuracy rate of 98-100% using eight partial indicators from the Beneish M-score model. Sormunen [28] further noted that the classification ability of certain financial ratios may diminish over time, suggesting the need for ongoing evaluation. In our literature review, we did not identify studies that predict inaccurate financial statements based on actual values of balance sheet items at the beginning and end of the period (without using ratio indicators). That is the research gap that this scientific paper aims to fill. The basis for determining inaccurate financial statements in government-owned enterprises is the analysis of auditors' findings and criticisms, as well as the written grounds for issuing opinions on the financial statements. According to our own research [5], the most common causes of irregularities in financial reporting of government-owned enterprises in FBiH include inadequate accounting estimates of accounts receivable from customers, inadequate valuation of fixed assets, inventory, and provisions. A range of methodologies have been proposed for predicting the auditor's opinion on financial statements. Sánchez-Serrano et al. [27] and Stanišić, Radojević & Stanić [30] both highlight the use of artificial neural networks and machine learning algorithms. Data mining can be an extremely useful tool for detecting irregularities within a large volume of data in financial statements. Large amounts of data often contain hidden patterns and trends that may indicate potential fraudulent activities. Therefore, data mining is used to extract knowledge from vast data sets in order to identify behavioral patterns that may indicate fraud. The study conducted by evaluated the capability of various Data Mining classification methods to identify companies that released fraudulent financial statements (FFS), with a particular emphasis on recognizing the key factors linked to such fraud. The study explored the application of Decision Trees, Artificial Neural Networks (ANN), and Bayesian Belief Networks as tools for detecting fraudulent financial reporting. The results demonstrated that the Bayesian Belief Network model achieved the best performance, successfully classifying 90.3% of the validation sample in a 10-fold cross-validation procedure. According to the research conducted by [26], utilized data mining techniques, including Multilayer Feed Forward Neural Network (MLFF), Support Vector Machines (SVM), Genetic Programming (GP), Group Method of Data Handling (GMDH), Logistic Regression (LR), and Probabilistic Neural Network (PNN), to identify companies involved in financial statement fraud. These techniques were tested on a dataset of 202 Chinese companies and compared with and without feature selection. Without feature selection, PNN outperformed all other techniques, while with feature selection, both GP and PNN achieved nearly equal accuracy, outperforming the others. According to the research conducted by [21] on financial statement fraud, utilizing data mining methods including logistic regression, decision trees (CART- C4.5. algoritam), and artificial neural networks (ANN), the results indicate that artificial neural networks and decision trees resulted in much more accurate classification compared to logistic regression. According to the research conducted by [29], the methods used in the fraud detection process were Linear Regression, Artificial Neural Networks (ANN), k-Nearest Neighbors (KNN), Support Vector Machines (SVM), Decision Stump, M5P Tree, Random Forest, and J48. The results from the experiments indicated that data mining methods were able to detect the fraud factors between the financial statements and the e-ledger. In this study, the Decision Stump Algorithm exhibited the best performance. In addition to the mentioned studies, numerous other research studies have focused on the effectiveness of different data mining techniques in detecting fraud in financial statements [17]. Tatusch et al. [31] introduced a modified version of DBSCAN, a density-based clustering algorithm, which outperformed prior methods in detecting restated financial statements. Ravisankar et al. [26] and Gill & Gupta [7] both found that probabilistic neural network (PNN) and neural network techniques were effective in identifying companies resorting to financial statement fraud. These studies collectively suggest that a combination of clustering and classification techniques, particularly those that incorporate temporal variation and financial ratios, can be effective in predicting inaccurate financial statements. All of these authors have investigated fraud detection in the profit sector. However, We were unable to identify significant scientific studies on detecting inaccurate financial statements in government-owned enterprises. Numerous authors have demonstrated the effectiveness of utilizing the Beneish M-Score model in practice, although they did not employ data mining techniques but rather relied solely on the mathematical formula of the model [10].

3. Research elaboration

3.1. Methodology

This paper examines the data extracted from the financial statements of Government-Owned Enterprises (GOEs) in the FBiH and the corresponding audit reports prepared by the Audit Office of the Institutions in FBiH. The study encompasses a time span of 16 years, from 2004 to 2019. It comprises a total of 200 financial statements and their associated audit reports, all of which were available at the time of conducting the study. Financial statements serve as the fundamental basis and starting point for analyzing business operations and assessing the condition of a company. They can also be regarded as confidential records generated by organizations that contain their financial transactions, including expenses, realized profits, income from loans, etc. [8]. Financial statements provide an indication of the organization's financial reality and also include management notes on business performance and projected future trends. Furthermore, inaccurate financial statements deceive users of financial reports by creating the impression that organizations are performing favorably. The task of auditing is to protect the interests of capital owners and provide a reliable information foundation for rational decision-making and management of state-owned companies. In accordance with the International Standard on Auditing (ISA) 240 (sections 2 and 3), misstatements in financial statements can occur due to either fraud or error (the International Standards of Supreme Audit Institutions-ISSAI does not specifically define inaccurate financial statements like ISA 240 does). The key differentiating factor between fraud and error lies in whether the underlying action that leads to the misstatement in financial statements is deliberate or unintentional.

While fraud is a broad legal concept, auditors, under the ISAs, focus on fraud that causes significant misstatements in financial statements. There are two types of intentional misstatements that are relevant to auditors: misstatements arising from fraudulent financial reporting and misstatements resulting from misappropriation of assets. Although auditors may suspect or, in rare cases, identify instances of fraud, they do not make legal determinations regarding the occurrence of fraud. Consequently, the term "misstatement" will be used going forward without explicitly specifying whether an error is intentional or not . Pursuant to International Standard on Auditing (ISA) 240 (sections 2 and 3), misstatements in the financial statements can arise from either fraud or error (International Standards of Supreme Audit Institutions-ISSAI does not specifically define inaccurate financial statements like ISA 240 does). The key difference between fraud and error lies in the intent behind the action that leads to inaccuracies in the financial statements—whether it is intentional or unintentional. Although the concept of fraud is broad in legal terms, under the International Standards on Auditing, the auditor focuses on frauds that cause material misstatements in the financial statements. There are two types of intentional misstatements relevant to the auditor: those arising from fraudulent financial reporting and those resulting from the misappropriation of assets [11]. Although the auditor may suspect or, in rare cases, identify the occurrence of fraud, the auditor does not make legal determinations of whether fraud has actually occurred. Therefore, the term misstatement shall be used onwards without specifying if error is intentional or not.

International Standard on Auditing (ISA) 705 (paragraph 2) defines three types of modified opinions: a qualified opinion, an adverse opinion, and a disclaimer of opinion. The decision regarding which type of modified opinion is appropriate depends on the nature of the matter causing the modification, i.e., whether the financial statements are materially misstated or, in cases where sufficient and appropriate audit evidence cannot be obtained, whether they could be materially misstated; and the auditor's judgment regarding the pervasiveness of the effects or possible effects of the issue on the financial statements [12]. Our model uses balance sheet items from financial statements as predictive attributes and the type of opinion on the financial statements as the target variable. As the input set of data for the model, we used 24 balance sheet positions: Opening and closing balances of accounts receivable (attribute code: A3, A1), Sales income for the current and previous accounting period (A4, A2), Operating expenses of the current and previous accounting periods (A6, A5), Opening and closing balances of operating assets (A11, A7), Opening and closing balances of Property, Plant, and Equipment (A12, A8), Opening and closing balances of short-term financial investments (A13, A9), Opening and closing balances of business assets (A14, A10), Opening and closing balances of Depreciation (A15, A16), Administrative expenses of the current and previous accounting period (A17, A18),

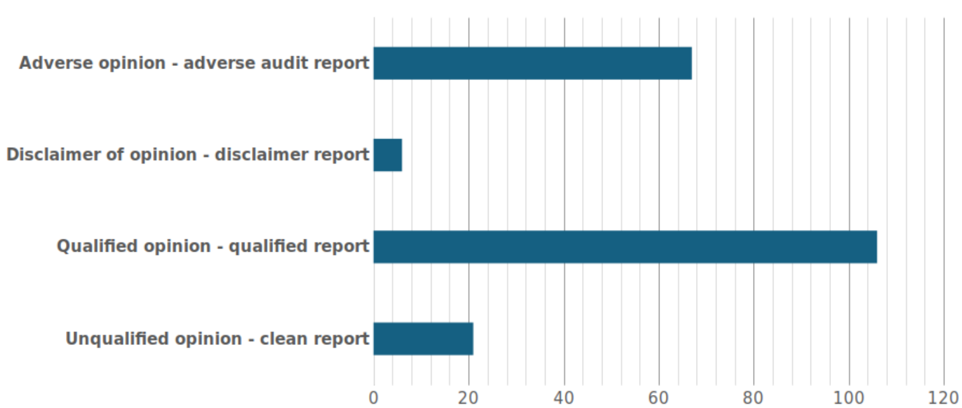

Figure 1: *Audit reports distribution for GOEs in FBiH.*

Opening and closing balances of short-term liabilities (A21, A19), Opening and closing balances of long-term liabilities (A22, A20), Business profit/loss for the current accounting period (A23) and Net cash flow from operating activities for the current accounting period (A24). These balance sheet items were selected based on the fact that auditors have predominantly identified irregularities in the valuation of these balance sheet positions as grounds for issuing qualified opinions. Auditors currently highlight non-compliance with the provisions of IFRS 9, IFRS 15, IAS 2, IAS 16, IAS 36, IAS 37, IAS 38, as well as cash flows from operating activities and business results in the financial statements of government-owned enterprises in FBiH. The output variable was the quality of financial statements measured relative to the audit report of state auditors. The results of the final audit reports for GOEs subject to the analysis are given in Figure 1. Number of enterprises is presented on the x-axis.

The output variable – audit report for GOEs in FBiH can be grouped as follows:

- As four categories or classes, so that audit report is a class, as presented in Table 1,
- As two classes coded as YES category – unqualified and qualified opinion, and NO category – adverse opinion and disclaimer of opinion, as presented in Table 2.

Class	**Audit report**	**Sample number**	**Percentage**
1	Unqualified opinion	21	10.50%
2	Qualified opinion	106	53.00%
3	Disclaimer of opinion	6	3.00%
4	Adverse opinion	67	33.50%
SUM		**200**	**100.00%**

Table 1: *Four classes, according to the final audit report.*

Class	**Assessment**	**Sample number**	**Percentage**
1	No	73	36.50%
2	Yes	127	63.50%
SUM		**200**	**100.00%**

Table 2: *Two classes, according to the final audit report.*

It is evident that prediction error in the first case would be much higher due to different distribution of the final audit report by classes. Hence, this research gave advantage to the second case. The enterprises are divided into two basic groups:

- The first group includes the enterprises whose financial statements were given unqualified or qualified opinion (127 enterprises).
- The second group includes the enterprises whose financial statements were given adverse of disclaimer of opinion (73 enterprises)

Such formulation of the output variable categorizes the problem as classification problem, where the aim of the model is to learn how to recognize the proper classification of the final audit report. The primary goal of prediction is to develop a model that derives insights about a specific characteristic of the dependent variable by utilizing a combination of independent variables. Predictive modeling involves determining the output variable for a constrained dataset, where the symbols represent the values of the output variable in particular instances. The choice of variables from the available dataset significantly influences the precision and accuracy of the resulting predictive models.

3.2. Data mining

Data mining, a field of knowledge discovery in databases [1], can be utilized for discovering financial frauds. Different types of data mining techniques can be employed for this purpose. Classification is a method used to evaluate and find a function that assigns items from a data set to predetermined classes of the output variable, based on the input variables' values [18]. Through classification, models can be created to classify unknown datasets into specific categories or classes [18]. The classification process typically involves the following steps:

- Selecting classifiers for implementing the classification algorithm.
- Choosing the class attribute (output variable).
- Dividing data sets into two: training data and test data.
- Training the classifiers on the training data set with known values of the class attribute.
- Testing the classifiers on the test data set with hidden values of the class attribute.

In the classification process, existing techniques are applied to evaluate the proposed prediction model using collected instances. In the case of selecting a classifier for datasets obtained from the financial statements of government-owned enterprises, the situation is quite clear. This dataset is very small; the procedure of conducting the experimentation phase is more difficult due to the fact that the data is dynamically changeable. Financial statements data are mostly of a numerical and categorical type, and since they are manually extracted from databases of financial statements, they require less cleaning in the preprocessing phase. Some applications of different classifiers on such data are described in the papers mentioned in the listed references. In the paper [15], the authors employed DT model for financial fraud detection in companies. Jan [14] used decision trees in combination with other data mining techniques to achieve a high accuracy rate of 90.83% in detecting financial statements fraud. Kirkos, Spathis & Manolopoulos [15] also explored the effectiveness of decision trees in detecting fraudulent financial statements, comparing their performance with other data mining techniques. These studies collectively highlight the potential of decision trees in detecting fraud in financial statements of government-owned enterprises. However, some recent approaches have been introduced for financial fraud detection [22]. Kırda & Özçelik [16] found KNN to be a highly effective classifier, achieving an accuracy rate of 91.73% in detecting financial statement fraud. Yao et. al. [36] also highlighted the effectiveness of KNN, particularly when combined with support vector machine (SVM) and stepwise regression, in detecting fraudulent financial statements. These studies collectively underscore the potential of KNN in detecting fraud in government-owned enterprises . A range of studies have demonstrated the effectiveness of Multilayer Perceptron (MLP) in detecting fraud in financial statements. Trigueiros & Sam [32] and Mubarek & Adali [24] both found that MLP, when used in conjunction with other machine learning techniques, outperformed traditional methods in fraud detection. Ravisankar et al. [26] and Kwon & Feroz [19] further support these findings, with Ravisankar [26] noting the superior performance of MLP in identifying companies engaged in financial statement fraud, and Kwon [19] reporting an 88% accuracy rate in predicting SEC investigation targets using MLP. These studies collectively highlight the potential of MLP in detecting fraud in financial statements, particularly in government-owned enterprises. For example, some studies have employed Bayesian networks to detect inaccurate financial statements. Deng [4] found the algorithm to be effective in this context, noting its potential for proactive fraud detection. Handoko, Wiyardi & Handoko [9] also found success in using the Beneish M-Score method, which includes variables related to financial statement manipulation, in detecting fraud in Indonesian government-owned enterprises. In this work, and in accordance with the above mention, the following algorithms were chosen: Decision Trees (J48) [14, 15], K-nearest neighbors [16, 36], Neural Network (MLP)

[8, 19, 24, 26, 32], and Bayesian network (Naive Bayes) [4, 9] for application on the training dataset. The following subsection offers an detailed overview of the machine learning techniques applied in this research to detect financial fraudulent activities. Decision Trees (DT) are one of the most well-known classification techniques often used to model data in the form of a tree structure. These algorithms establish relationships between input features and outputs using a tree-like structure, named for its resemblance to an inverted tree. The name "decision tree" derives from the fact that it resembles an inverted tree. The most commonly used and widely recognized decision tree algorithm is C4.5, and its implementation in the Weka software tool is known as the J48 algorithm. The J48 (C4.5) algorithm [25] presents an extension of Professor Ross Quinlan's earlier ID3 algorithm, and is known for its exceptionally high accuracy. The advantage of the J48 algorithm is the ability to work with numerical and categorized data [34], in addition, it is easy to implement and effectively deals with noise and missing values [23], and it has the ability to display results graphically. Basic construction of J48 algorithm uses a method known as divide and conquer to display output [34, 23]:

- Select the dataset as input for the rule-making process.
- Calculate the normalized information gain for each attribute.
- Choose the attribute with the maximum information gain as the best attribute. This attribute becomes the root node and corresponding to the best predictor.
- Repeat the above step until a stopping criterion is met, calculating the information gain for each attribute and adding that attribute as a child node.

Just as a tree starts from the root, branches into individual branches, and ends in leaves, decision trees use branches to represent decision paths, with the final outcome represented by the leaves. The final result is a tree with decision nodes and leaves. Each decision node has two or more divisions, while each leaf represents an outcome or decision.

K-nearest neighbors (KNN) algorithm represents a straightforward and easily understandable supervised learning method commonly applied in both classification and regression problems . It belongs to a group of algorithms known as instance-based learning, sometimes referred to as 'lazy learning methods' because the processing of the training data is delayed until a test instance needs to be classified. KNN operates on the principle that objects with similar characteristics tend to belong to the same class or have similar output values. The main goal of KNN is to group n objects into k groups (classes) based on their attributes or features. When a testing example is considered, it is placed in an n-dimensional metric space of attribute values. The algorithm then determines the distance between the testing example and all training examples in this space. For classification, the most popular classification among the k nearest training examples is the target estimation for the classifier. To define 'nearest,' various metrics can be used, including the standard Euclidean distance and Hamming distance, with Euclidean distance being commonly used. If k is greater than 1, those closer to the testing example will have a greater weight in the classification. The procedure starts with an initial division of the set items into a selected number of groups. The distance between every object and every group is determined [22], and objects are located into groups closest to them based on given characteristics. After joining an object to a group, the centroid of the group is recalculated. The distance of every object from the group centroid is recalculated, and the distribution of objects among groups continues until the selected function of the criterion suggests otherwise. Because of all the above, the KNN algorithm is easy to implement and understand, and is more suitable for small data sets. However, for large datasets, it can be computationally demanding because it requires calculating the distances to all instances in the training set for each new instance.

Multilayer Perceptron (MLP) algorithm is one of the most frequently used and known neural network. The network consists of a set of perceptrons that make an input layer, one or more hidden layers of process elements and an output layer [34]. MLP is particularly suitable for the approximation of classification functions (when we have little knowledge on the ratio between input and output attributes) that map the example determined by the attribute value vector into one or more classes.

Multilayer perceptrons (MLPs) utilizes a feed-forward structure where signals flow in one direction, from the input layer to the output layer without any feedback loops, providing a nonlinear input-output mapping. The basic nonlinear mappings and summation layers are determined by the structure of the MLP. Layers that are not directly connected to the environment are called hidden layers. In this research, we chose an MLP with one hidden layer, using a hyperbolic tangent activation function $\tanh(a) = \frac{e^a - e^{-a}}{e^a + e^{-a}}$ for the hidden neurons, and a logistic sigmoid function as the output activation function. However, any other activation function, such as a threshold function, could also be used. The MLP model can be described as follows:

$$f(x, w) = g\left(\sum_{j=0}^{M} w_j^{(2)} \tanh\left(w_{ji}^{(1)} x_i\right)\right), \tag{1}$$

where w contains the weights and the bias parameter and i and j stand for input and hidden units, respectively. Since the logistic function's output value can be interpreted as a probability and ranges from 0 to 1, it was selected as the output activation function in this case.

Indeed, the Multilayer Perceptron (MLP) algorithm has shown significant potential for use in the financial sector, particularly in detecting Financial Statement Fraud [8]. MLP's ability to handle complex patterns and nonlinear relationships in data makes it well-suited for identifying irregularities and anomalies in financial statements. By analyzing historical financial data and patterns, MLP can learn to detect suspicious activities and behaviors that may indicate fraudulent practices.

Naive Bayesian (NB) algorithm is widely used in machine learning technique for classification and has shown effectiveness in various domains. Naive Bayes is a simple classification method that predicts membership probabilities for each class based on Bayes' theorem [4, 34]. It operates under the assumption that the attributes are statistically independent of each other. Naive Bayes classifier uses equation to classify the value of the target variable y based on the assumption that the attribute variables, $x_1, \ldots, x_p$, are independent of each other:

$$y_{MAP} \approx \mathrm{argmax}_{y \in Y} p(y) \prod_{i=1}^{p} P(x_i|y). \tag{2}$$

It calculates the probability of a data instance belonging to a particular class by combining prior probabilities and conditional probabilities. By using these probabilities, NB can make predictions and classify new data instances into the most probable class [34]. The term "naïve" refers to its simplification of the problem by relying on two key assumptions: it assumes that prognostic attributes are conditionally independent given the known classification, and it assumes there are no hidden attributes that could influence the prediction process. This approach is a promising method for probabilistic knowledge detection and enables an efficient algorithm for financial fraud detection.

To assess the robustness of classifiers [1], we used triple cross validation, that is we randomly divided a set of data into three subsets of equal size. Two subsets were used for training and one was used for cross checking the validity. This procedure was executed three times so that each subset was tested once. The test results stand for the given average value of each of the cross checking. For the implementation of the selected classifiers, we used the Weka 3.8.5 data mining tool.

4. Results and discussions

In the previous stages of developing the mathematical model for detecting fraudulent financial statements in the government-owned enterprises of FBiH, it was essential to identify key variables and indicators that would enhance predictive capability. After collecting and extracting the data, we gained insights into their structure and informative value to prepare for the implementation of data mining algorithms. To achieve this, we utilized simple statistical techniques and visualization tools. Histograms were created to visualize the distribution of nominal attribute values, numerical attribute distributions were examined, and graphical presentations were generated to analyze attribute values relative to the class or other attributes. These techniques helped in identifying unexpected data patterns, indefinite attribute values, duplicate data, and other anomalies. In order to gain more detailed insight into the importance of input variables, the values of input attributes relative to the output attribute were assessed, along with analyzing the influence of individual input variables on the model relative to the output variable. The aim of this assessment and attribute selection was to extract irrelevant and redundant attributes from the training data set. Data filtering included techniques to assess attribute values based on heuristics derived from general data features. Filtering methods proved to be a more practical solution for intelligent data analysis, as they significantly shortened attribute selection and assessment processes. Their independence from machine learning algorithms allowed for their implementation alongside any data modeling technique. The filtering methods used were InfoGain and GainRatio with the Ranker search method. InfoGain assesses the value of an attribute by measuring its information gain relative to class. GainRatio assesses the value of an attribute by measuring relative information gain to the class. The attributes assessed less than 0.01 need to be excluded from the analyzed data set. The results of the assessment and ranking the attributes based on their individual values are given in Table 3.

Table 3 shows that out of 24 balance sheet positions, seven balance sheet positions best describe misstatement of financial statements of GOEs in FBiH. They are: opening entry of accounts receivable (A3), profit (loss) at the end of the period (A23), operating assets at the end of the period (A7), accounts receivable at the end of the period (A1), opening entry of operating assets (A11), short term financial investments at the end of the period (A9), and opening entry of short-term financial investments (A13).

If we observe the audit bases used for giving opinions on financial statements, it can be concluded that the results largely match. Namely, as the auditors found, the most frequent error in the financial statements is that public procurement procedures are not in line with the Law on Public Procurements. This directly affects financial statements through booking procurement of fixed assets, services or consumable goods. This objection given by the auditors is evident in 42 out of 200 enterprises. The error registered in 35 enterprises refers to accounts receivable not being value adjusted. What follows is the error identified in 34 enterprises that do not measure reimbursable value of fixed assets. Stocks not being properly showed is an error placed fourth, registered in 26 companies, same as the error of reserves not being properly shown.

Table 3 indicates that the attributes A03, A23, A07, A01 and A09 have the highest ranking while A11 and A13 have the lowest ranking. Since the remaining attributes have the values below 0.01, we will have the fourth set of input data to test their predictive capability. This fourth set of input data includes only the attributes (seven balance sheet positions with informative value).

The algorithms Decision Trees (J48), KNN, Neural network (MLP) and Naive Bayes for creating prediction model were used in this research (Table 4). The model estimate was made by a triple cross validation method.

Table 4, in section B, shows that for the set of data with seven attributes and informative value above 0.01, using the MLP algorithm, the most accurate prediction model was created.

All atributes			Balance sheet positions
ATTRIBUTES	**InfoGain**	**GainRatio**	**Description**
A03	0.1665	0.1305	Accounts receivable t-1
A23	0.1050	0.1758	Business profit/loss t
A07	0.0892	0.1271	Operating assets t
A01	0.0649	0.1165	Accounts receivable t
A11	0.0639	0.0885	Operating assets t-1
A09	0.0589	0.1404	Short-term financial investments t
A13	0.0516	0.0678	Short-term financial investments t-1
A02	0	0	Sales income t
A04	0	0	Sales income t-1
A05	0	0	Operating expenses t-1
A06	0	0	Operating expenses t
A08	0	0	Property, plant and equipment t
A10	0	0	Business assets t
A12	0	0	Property, plant and equipment t-1
A14	0	0	Business assets t-1
A15	0	0	Depreciation t-1
A16	0	0	Depreciation t
A17	0	0	Administrative expenses t
A18	0	0	Administrative expenses t-1
A19	0	0	Short-term liabilities t
A20	0	0	Long-term liabilities t
A21	0	0	Short-term liabilities t-1
A22	0	0	Long-term liabilities t-1
A24	0	0	Net cash flow from operating activities t

Table 3: *Results of evaluating and ranking attributes.*

The MLP algorithm generated the model with 76% correctly classified items (CCI), precision of 75.6% (0.756), and classification above the segment of the ROC curve (0.754 > 0.5). Predictive capability for the first set of data A, made of 24 balance sheet positions is between 57.5% and 71%, depending on the implementation of different classification algorithms.

Ultimately, the results indicate that the MLP algorithm provides the most accurate model for the set of 7 attributes (A3, A23, A7, A1, A11, A9 and A13) with high precision and classification above the ROC curve segment. This suggests that MLP was an efficient choice for data analysis, considering the selected attributes and learning methodology.

It is important to emphasize that the implementation of the MLP neural network and the validation by means of dividing the data set in such a way that two thirds are used for the creation of the model and one third for its validation, result in 4% on average better prediction indicators for all sets of input data. This leads to the conclusion that a larger set of input data might prove better results of predictive capability, using the same methodology. A small data set is the main limitation of this research.

Limitations of this study relate to a relatively small research sample. A sample of 200 financial statements is not large enough for the applied data mining methodology. However, collecting a larger volume of historical data would not be relevant due to changes and adoption of new international accounting and financial reporting standards.

DATA SET	EVALUATION CRITERIA	CLASSIFIERS			
		J48	KNN	MLP	BayesNet
A	Timing to build model (in Sec)	0	0	0.41	0
	Correctly classified instances	142	141	139	115
	Incorrectly classified instances	58	59	61	85
	Prediction accuracy	71	70.5	69.5	57.5
	Kappa statistic	0.3316	0.397	0.3552	0.1264
	Mean absolute error (MAE)	0.3163	0.2916	0.3403	0.4175
	Root mean squared error (RMSE)	0.5021	0.4626	0.4445	0.4887
	Relative absolute error (RAE)	68.16%	62.82%	73.33%	89.96%
	Root relative squared error (RRSE)	104.29%	96.07%	92.32%	101.51%
B	Timing to build model (in Sec)	0	0	0.08	0
	Correctly classified instances	129	124	152	115
	Incorrectly classified instances	71	76	48	85
	Prediction accuracy	64.5	62	76	57.5
	Kappa statistic	0.1635	0.2501	0.4536	0.1264
	Mean absolute error (MAE)	0.4051	0.3685	0.3584	0.4175
	Root mean squared error (RMSE)	0.4997	0.5313	0.4348	0.4887
	Relative absolute error (RAE)	87.29%	79.39%	77.23%	89.96%
	Root relative squared error (RRSE)	103.77%	110.34%	90.30%	101.51%

Table 4: *The performances obtained by the previous classification algorithms using the 24 selected attributes (A) and the seven selected attributes (B).*

5. Conclusions

Aiming to provide a specific solution to the problem of detecting misstatements in financial statements of GOEs in FBiH whose majority capital is mainly owned by the federal, cantonal and local levels of authority, we implemented four different algorithms of data mining (J48, KNN, MLP and BayesNet) on four sets od input data. Using filtering methods InfoGain and GainRatio with the Ranker search method, we established that seven balance sheet positions have the largest significance for future development of a mathematical model of prediction of misstatements in GOEs' financial statements. They are: opening entry of accounts receivable, profit (loss) at the end of the period, operating assets at the end of the period, accounts receivable at the end of the period, opening entry of operating assets, short term financial investments at the end of the period, and opening entry of short-term financial investments. These seven balance sheet positions have as many as 76% of predictive capability, i.e., correct classification of financial statements, with the estimation of the model by a triple cross validation method. If the validation is set in such a way that two thirds of data are used for the creation of the model and one third for its validation, we get even better indicators of predictive value. This points to the fact that a larger data set might prove higher predictive capability. The relatively small sample of 200 financial statements from government-owned enterprises represents a limitation of the conducted research.

This research also contributes to filling the literature gap regarding the attributes used to detect inaccurate financial statements in government-owned enterprises. Unlike other similar studies, this work uses balance sheet positions from financial position reports of enterprises instead of financial ratios as attributes.

Future development of mathematical model for predicting misstatements of financial statements made by GOEs in FBiH should focus on seven identified balance sheet positions. This can serve as the recommendation for future research in this field. The benefits of development and application of this model by the public, Financial-intelligence Agency of FBiH (FIA) and the Audit Office of the Institutions in FBiH can be identified as follows:

- at the point of financial statement submission, the state agency would be able to detect GOEs with high probability of misstatements in their financial statements;
- the Audit Office of the Institutions in FBiH would spend less resources as they would have a focused action strategy and audit those GOEs with high probability of misstatements in their financial statements;
- the community would be more efficient in detecting misstatements in fiancial statements and consequently spend less resources and have a better control of the process.

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Measuring trade facilitation for the emerging seven countries (E7) using multi-criteria decision-making methods

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Abstract. Countries provide opportunities for traders not only in imports but also in export performance by simplification and harmonization of documents in order to facilitate certain policy areas that have a tremendous impact on trade volumes. The OECD has established various indicators at different levels and compared countries to measure the trade facilitation between countries. This study aims to compare the procedures applied by the emerging seven countries (E7- Brazil, China, India, Indonesia, Mexico, Russia, and Türkiye) using trade facilitation indicators (TFIs) and multi-criteria decision-making methods. We employ MEREC, WENSLO, ENTROPY, LOPCOW, CVM, CRITIC, ANGLE, and GINI methods to determine the weights of 11 TFIs identified by the OECD. The most suitable weighting method is established through multi-dimensional analysis. E7 countries are then evaluated using the MABAC and ARTASI methods, with a sensitivity analysis comparing results against 38 OECD member countries. In terms of performance across all TFIs, Russia ranks first (MABAC-0.253, ARTASI-1.821), Mexico second (MABAC-0.186, ARTASI-1.802), and Türkiye third (MABAC-0.117, ARTASI-1.731).

Keywords: ANGLE, emerging seven, OECD, MABAC, trade facilitation

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1. Introduction

The value of global merchandise trade (exports), segmented by product groups, showed a continuous increase from 2020 to 2022: in 2020 (-7,4 percent: 17.65 trillion USD), 2021 (+26.5: 22.32 trillion USD) and 2022 (+12.2: 24.92 trillion USD). However, there was a partial decrease in 2023 (-4.5: 23.78 trillion USD) compared to the previous year according to WTO (World Trade Organization) data. Similarly, global merchandise imports also increased continuously during this period: in 2020 (-7.9: 17.88 trillion USD), 2021 (+26.4: 22.59 trillion USD) and 2022 (+14.3: 25.7 trillion USD). Nonetheless there was a partial decrease in 2023 (-5.7: 24.23 trillion USD) compared to the previous year [30]. According to the WTO's 2024 annual report, trade in goods is expected to decline in 2023 as the continuing effects of inflation and high energy prices affect demand for trade-intensive manufactured goods. Conversely, trade in commercial services continues to expand. The WTO anticipates that trade in goods is expected to recover gradually in 2024 and 2025 as low inflation increases real household incomes and boosts import demand [31].

The world economy has undergone a rapid transformation in recent years, leading to an increase in international trade necessitates that necessitates removal of trade barriers. The

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OECD measures the economic and trade impact of specific trade facilitation initiatives for countries. Research indicates that, integrating these trade facilitation indicators (TFIs), could potentially lower trade costs by nearly 10 percent [15]. In WTO negotiations on trade facilitation, the costs to developing countries of implementing trade facilitation measures have been a central concern. The total capital expenditure required to introduce trade facilitation measures in the reviewed countries ranged from €3.5 to €19 million, while annual operating costs -both direct or indirect- did not exceed €2.5 million in any of these countries [17]. Another study found that cost reduction potentials to reach approximately 14.5 percent of trade costs for low-income countries, 15.5 percent for lower-middle-income countries, and 13.2 percent for upper-middle-income countries [16].

Trade facilitation refers to policies and measures aimed at reducing trade costs by increasing effectiveness at all stages of the international trade chain. Regardless of its definition and scope, economic analyses of trade facilitation generally draw on the concept of lowering trade transaction costs and aim to assess the benefits of trade facilitation measures by increasing efficiency in various policy areas affecting the movement of goods [15, 16]. Wilson et al. [28] explained that trade facilitation typically means improving efficiency in administration, procedures, and logistics at ports and customs. It also includes streamlined regulatory conditions, deeper harmonization of standards, and compliance with international regulations. The traditional perspective on trade facilitation aims to enhance border and transit management procedures and their implementation, thereby removing barriers to trade in goods at the borders. Trade facilitation can efficiently decrease the cost of international trade, ease trade conflicts, encourage trade between countries, and assist countries in reducing overall trade costs. Eventually, trade facilitation involves the simplification, harmonization, standardization and modernization of trade procedures- an agenda item of many customs-related activities aimed at lowering trade transaction costs at the interface between business and government. While there is no universally accepted definition for trade facilitation, in a narrow sense, trade facilitation efforts may handle the logistics of moving goods through ports or improving transportation efficiency associated with cross-border trade [27].

This study makes important contributions to the literature through several unique aspects. It is the first to examine trade facilitation across countries using numerical evidence, specifically TFIs are investigated for E7 countries while utilizing comprehensive data from OECD countries for comparison. The methods and analyses employed in this study are quite comprehensive, focusing on identifying the most appropriate decision-making method rather than relying on a single approach -an important consideration in the field of decision-making. Multiple comparisons of multiple methods have been successfully implemented with a high level of robustness. This study aims to compare the procedures applied by emerging countries for TFIs using multi-criteria decision-making (MCDM) methods. Indicators are analyzed through multiple decision methods and multi-dimensional thinking. Specifically, this study utilizes MEREC, WENSLO, ENTROPY, LOPCOW, CVM, CRITIC, ANGLE, and GINI methods to analyze the indicators more effectively. Multi-dimensional thinking comes into play here to decide which method is most appropriate. Additionally, the MABAC and ARTASI methods are used to evaluate different countries within defined limits. The MABAC method provides robust decision support by calculating the "border approximation area" for each alternative. It effectively reflects the impact of criteria weights on the overall decision, offering more accurate results when the priority of criteria changes significantly. The ARTASI method is effective in decision problems where similarity comparisons are key, using "approximate ratios and total area-based similarity indices". The ARTASI method can adjust data of different sizes using various areas or ranges, such as (1, 10), (1, 100), (0, 1), (0, 2), or (1, 1000). The underlying calculation logic of these methods is similar: both present calculations within a defined limit range. Given the detailed calculation stages of multiple mathematical methods, it is crucial that findings are presented accurately, and that careful calculations are made. The remainder of the paper is organized as follows: the second section reviews the relevant literature; the third section explains the methods and materials used in the study; the results are presented in the fourth section; a sensitivity analysis is conducted in the fifth section; the sixth section discusses the findings and their managerial implications; and finally, the concluding remarks and future perspectives of the study are provided in the last section.

2. Literature review

In international trade, tariffs have been remarkably reduced through a combination of multilateral, regional and unilateral efforts, and on the other hand, countries that have actively pursued trade facilitation have gained from lower trade costs while participating in the ongoing multilateral negotiations. This indicates that trade facilitation is particularly important for development prospects as two conflicting dynamics in today's international trading system [21]. The new trade issues introduced at the multilateral trade negotiations in December 1996 at the Singapore Ministerial meeting: labeled as "Singapore Issues", include competition policy, investment, transparency in government procurement, and trade facilitation [27]. The impact of the TFIs is important not only in specifications regarding emerging countries as importers, but also when emerging countries are exporting to the rest of the countries (emerging and developed economies) in the overall sample [16]. Empirical research on trade facilitation faces three challenges: defining and measuring trade facilitation; choosing a modeling methodology to estimate the significance of trade facilitation; and designing a scenario to evaluate the impact of improved trade facilitation on trade flows [28].

Wilson et al. [29] developed four measures of trade facilitation: customs environment, port efficiency, regulatory environment, service sector infrastructure, and included them in a gravity model for trade flows: these set of indicators offer more details to policymakers about what kind of trade facilitation actions might provide the largest improvements in terms of increasing trade flow. Shepherd and Wilson [21] examined trade facilitation -including port infrastructure, customs administration air transport infrastructure, and services sector development- within ASEAN member countries and found that import and export costs vary significantly across member countries, from very low to moderately high levels, and tariff and non-tariff barriers are low to moderate. Sénquiz-Díaz [23] analyzed the impacts of tariff-related and trade barriers, trade facilitation, logistics, and trade using a sample of 80 countries: the results show that transportation infrastructure has a substantial direct impact on trade facilitation and logistics has a comparatively smaller direct effect.

Trade facilitation measures have become essential tools for creating a better trading environment. The international community recognizes that for many low-income countries, improved market access to industrialized countries is insufficient unless their trading capabilities are also enhanced. Efficient trade facilitation, such as improving the efficiency of border procedures by investing in infrastructure and human resources, can help reduce trade transaction costs, thereby reduce the spread between domestic and international prices, benefiting both consumers and producers [14] The prime trade drivers of the growth process are the Emerging Seven (hereafter E7) countries rather than other countries. The remarkable growth experienced by the E7 over the last two decades has allowed it to catch up with the G7, and such a high growth indicator also makes the E7 sensitive to trade flows due to rising demand for goods.

3. Material and methods

The OECD developed a set of TFIs that specify areas of reforms to help governments prioritize actions and mobilize more targeted technical assistance and capacity-building efforts for developing countries. The TFIs set by the OECD also help countries identifying their strengths and weaknesses in trade facilitation. Initially, the TFIs were designed to measure seven different categories of trade facilitation efforts: port logistics, customs procedures, own regulatory environment, standards harmonization, business mobility, e-business activity, and administrative transparency and professionalism, which consisted of surveys and economic evidence on trade facilitation [27]. Subsequently, twelve TFIs were constructed to correspond to the main policy areas under negotiation at the WTO (Moïsé et al., 2011:5). The TFIs provide a structured overview of the trade facilitation policy environment and closely follow the structure in over 160 economies including 155 measures in total [24]. The OECD TFIs measure the extent to which countries implement trade facilitation measures and their performance compared to others. The indicators take values between 0 to 2, where 2 indicates the best possible performance. The TFIs currently applied by the OECD are presented in Table 1.

TFIs	Indicators	Explanations
TFIs1	Information availability	-availability of access to applicable and published legislation information;
TFIs2	Involvement of the trade community	-represents the participation of the entire trading community and consultations;
TFIs3	Advance rulings	-accessibility of rulings to the general trade, length of time a ruling is in effect, timeliness of issuance;
TFIs4	Appeal procedures	-the transparency, fairness, accessibility, timeliness, and effectiveness of the applicable rules and of outcomes;
TFIs5	Fees and charges	-availability of publicly available information about applicable fees and charges;
TFIs6	Formalities - documents	-harmonization of trade documents, simplification of documentary requirements, the use of copies, and the reduction of the number and complexity of required documentation;
TFIs7	Formalities - automation	-automated procedures, electronic interchange of documents (EDI), the application of risk management procedures;
TFIs8	Formalities - procedures	-single windows, pre-arrival processing, physical inspections, post-clearance audits, separation of release from clearance, authorized traders;
TFIs9	Internal border agency co-operation	-one-time documentary controls and coordinated physical inspections, average clearance times;
TFIs10	External border agency co-operation	-extensive co-operation and exchange programmes with neighboring and third countries;
TFIs11	Governance and impartiality	-good governance characteristics, clearly established and transparent structures and functions, ethics policy, a code of conduct, internal audits and transparent provisions for financing and sanctions.

Table 1: *Set of trade facilitation indicators.*

The relative economic and trade impact of specific trade facilitation measures offers countries with significant gains. This study examines trade facilitation as a means to enhance trade in emerging countries, enabling them to advance move rapidly in their development. The Emerging Seven (E7) is not an actual forum nor alliance, but merely a concept of the economic potential of emerging countries versus developed economies. The E7 countries consist of Brazil, China, India, Indonesia, Mexico, Russia, and Türkiye. The advantages offered by emerging countries are closely monitored around the globe. Representing the world's largest emerging economies, the E7 countries hold an important position among emerging markets in terms of size and potential, population, resources and production, market size, investment attractiveness, technological and industrial developments, and have a major impact on global economic dynamics. By simplifying and harmonizing processes, trade facilitation enhances trade by providing convenience to all parties involved. Trade facilitation is achieved through a variety of tools and varies from country to country. This study compares the trade facilitation performance of the seven emerging countries using indicators determined by the OECD. Making comparisons based on a robust mathematical structure is essential for stakeholders who will benefit from these results when measuring multiple indicators across different countries [11]. A structured framework that considers various criteria and alternatives is critical for decision-making. MCDM methods provide a structured and systematic approach to evaluating decisions with multiple conflicting criteria, challenging traditional analytical methods. Depending on the objective of decision-making and application area, each MCDM method has different specifications and calculation stages [20, 10, 11]. This study uses MEREC, WENSLO, ENTROPY, LOPCOW, CVM, CRITIC, ANGLE, and GINI methods to analyze the indicators effectively. Additionally, the MABAC and ARTASI methods are used to rank different countries. Instead of explaining all mathematical methods one by one, citing the studies from which they emerged and the solution stages provides a significant accelerating effect for this study and allows a greater focus on trade facilitation.

The MEREC method was introduced to the literature as a new objective method for decision-making. The WENSLO method was introduced to the literature by [18], is a novel weighting method. The ENTROPY method has been frequently used as the oldest weighting method in the MCDM area since it was introduced to the literature as information uncertainty measure by Shannon [10]. The LOPCOW method is a relatively new method introduced to the literature with the idea of obtaining reasonable weights by evaluating data of different dimensions together and reducing the gap between the weights of the criteria. The Coefficient of Variation (CVM) method is used in the MCDM field to find the criteria weights as a statistical measure of the distribution of data points around the mean in a data-series. The CVM method represents the ratio of the standard deviation to the mean. Even if the means are very different from each other, it can be used to find the criteria weights and a useful statistic to measure the difference between criteria [22, 8]. The CRITIC method was introduced with an approach that considers the correlations between the criteria in order to find the criteria weights objectively. The CRITIC method finds the criteria weights closer to each other and objectively based on real data in its calculations. The ANGLE/Angular method by [22] uses the weights of objective criteria as references to measure the angles between itself and other attributes geometrically. The GINI method [12] was used as the Gini coefficient-based criterion weight determination method. The MABAC method calculates the values of the criterion functions and ranks the alternatives by their distance to the border approximation area. The ARTASI method, introduced by [19], ranks alternatives based on distance measurement at standardized intervals. Recent studies in the literature using the 10 different MCDM methods mentioned in this study are presented in Table 2.

Although the MCDM family does not have a very long history, it has come a long way in recent times. New methods are introduced to the literature and practitioners/academics make a choice among them by the most suitable one for their purposes.

4. Results

The TFIs are used to effectively compare selected countries. However, determining the weights of these indicators is an important aspect of the decision problem. There are multiple methods in

Method	Applications of the study, Researcher(s)
MEREC	Evaluating the social factors within the circular economy [7]
MEREC	Ranking the Asian countries for quality of life index [3]
WENSLO	Measuring sustainable brand equity performance [5]
ENTROPY	Selection of an optimal renewable energy source [4]
LOPCOW	Comparison of indexed business journals [8]
LOPCOW	Ranking the Asian countries for quality of life index [3]
CVM	Green electricity generation assessment [25]
CVM	Comparison of indexed business journals [8]
CRITIC	Evaluating the social factors within the circular economy [7]
CRITIC	A hybrid MCDM model to improve decision-making stability, reliability [2]
ANGLE	A Python library for determining criteria significance [1]
ANGLE	Green electricity generation assessment [25]
ANGLE	The most advantageous renewable energy storage devices [9]
GINI	Green electricity generation assessment [25]
GINI	A Python library for determining criteria significance [1]
MABAC	Selection of an optimal renewable energy source [4]
MABAC	A hybrid MCDM model to improve decision-making stability, reliability [2]
MABAC	Evaluation of urban quality improvement [13]
ARTASI	Website performance analysis [6]

Table 2: *Studies using objective methods.*

the MCDM literature, each with its own calculation stages [26]. When performing calculations with MCDM methods, an initial decision matrix is required, as shown in Table 3.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
Brazil	1.667	1.500	1.545	1.333	1.846	1.778	1.538	1.607	1.273	1.182	1.889
China	1.619	1.857	1.857	1.667	1.923	1.556	1.538	1.556	1.455	0.900	1.875
Indonesia	1.524	1.571	1.400	1.556	1.538	1.375	1.200	1.630	1.400	1.000	1.556
India	1.905	1.429	1.300	1.417	1.769	1.556	1.692	1.515	1.909	1.000	1.750
Mexico	1.571	1.750	1.500	1.364	1.846	1.625	2.000	1.600	1.636	1.545	1.889
Russia	1.905	1.875	1.700	1.462	1.857	1.750	1.769	1.594	1.636	1.182	2.000
Turkey	1.667	1.875	1.364	1.455	1.692	1.875	1.667	1.645	1.800	1.091	1.889

Table 3: *An initial overview of trade facilitations for E7 countries.*

This study employs multiple methods, and the weights for TFIs are determined using objective methods via their calculation stages outlined in Table 4.

Considering the calculations from the WENSLO (0.263), ENTROPY (0.231), ANGLE (0.158) and GINI (0.153) methods, the "w10-external border agency co-operation" indicator was found to be more important than the others.

In these same calculations, the "w7-Formalities – automation" indicator is found to be the second most important indicator, and then in the MEREC method, the most important (0.167) indicator. Both of these indicators (w10-0.143 and w7-0.131) also occupy the top two positions in the "mean" rankings, where the criteria weights are combined. Therefore, it is thought that better results can be achieved by averaging the criteria weights [7].

The linear relationship between the values found by the methods can be examined using correlation analysis. Analyzing the Pearson correlations between the criteria reveals that the high-

	MEREC	WENSLO	ENTROPY	LOPCOW	CVM	CRITIC	ANGLE	GINI	mean
w1	0.058	0.059	0.056	0.062	0.069	0.113	0.077	0.076	0.071
w2	0.093	0.065	0.090	0.085	0.086	0.091	0.096	0.097	0.088
w3	0.086	0.125	0.115	0.063	0.070	0.091	0.111	0.114	0.097
w4	0.053	0.042	0.042	0.065	0.071	0.123	0.067	0.068	0.066
w5	0.082	0.041	0.038	0.123	0.116	0.076	0.063	0.061	0.075
w6	0.099	0.063	0.074	0.102	0.099	0.075	0.088	0.090	0.086
w7	0.167	0.174	0.163	0.113	0.108	0.066	0.129	0.129	0.131
w8	0.028	0.006	0.005	0.114	0.108	0.114	0.024	0.024	0.053
w9	0.121	0.118	0.141	0.086	0.086	0.103	0.121	0.127	0.113
w10	0.121	0.263	0.231	0.059	0.067	0.089	0.158	0.153	0.143
w11	0.092	0.044	0.044	0.128	0.121	0.059	0.067	0.062	0.077
MEREC	1								
WENSLO	0.731	1							
ENTROPY	0.788	0.981	1						
LOPCOW	0.109	-0.407	-0.402	1					
CVM	0.103	-0.404	-0.404	1.000	1				
CRITIC	-0.641	-0.235	-0.235	-0.617	-0.623	1			
ANGLE	0.818	0.938	0.976	-0.440	-0.442	-0.275	1		
GINI	0.814	0.917	0.966	-0.455	-0.459	-0.240	0.996	1	
mean	0.887	0.957	0.981	-0.236	-0.239	-0.380	0.962	0.951	1

Table 4: *Weights and correlations of trade facilitation indicators.*

est correlations are between GINI-ANGLE (0.996): ENTROPY-WENSLO and ENTROPY-Mean (0.981): ENTROPY-ANGLE (0.976): ENTROPY-GINI (0.966): ANGLE-Mean (0.962). These correlations are notably high (See Table 3).

However, knowing which method provides the best results when multiple methods are used can be challenging for decision-makers and practitioners. In such cases, the decision maker's subjective judgment emerges and a method is selected based on the structure of the problem. On the other hand, when more than one method is used, taking their averages can make things easier, but also determining the most appropriate method via distance between the methods or their distance from the mean can make things easier. The distances of the eight methods to the mean were analyzed, considering that multidimensional scaling analysis could be useful, as shown in Figure 1.

Although the GINI method also appears to be close, the ANGLE method is the most suitable for this decision problem. Multidimensional analysis, as shown in Figure 1, along with correlation analysis, indicates that the ANGLE method provided results that were more similar to the mean compared to the other methods. Once criterion weights are found using various methods, the next stage in the decision problem involves ranking the alternatives. In this case, the ANGLE weighted MABAC-ARTASI methods are used to rank the alternatives, as shown in Table 5.

	ANGLE- MABAC		ANGLE- ARTASI	
Brazil	-0.050	6	1.622	6
China	0.074	4	1.681	4
Indonesia	-0.290	7	1.440	7
India	-0.038	5	1.635	5
Mexico	0.186	2	1.802	2
Russia	0.253	1	1.821	1
Türkiye	0.117	3	1.731	3

Table 5: *Ranking of the E7 countries.*

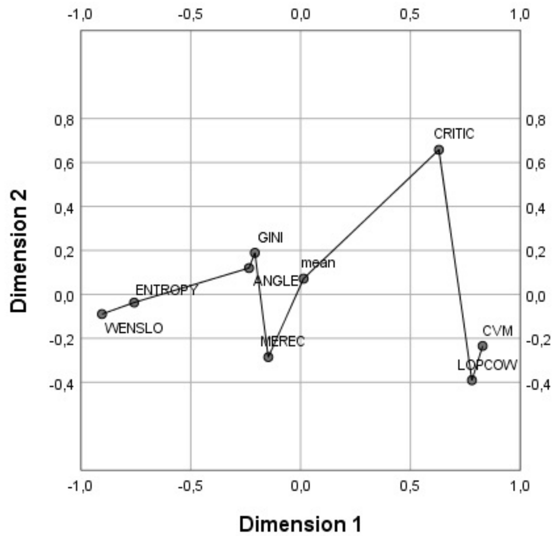

Figure 1: *Results of the multidimensional scale analysis.*

In both methods (MABAC and ARTASI), Russia ranks first, followed by Mexico, Türkiye, China, India, Brazil, and Indonesia (see Table 4). When the TFIs criteria values are compared using MCDM methods, Russia is found to have the most favorable values among the E7 countries. Russia stands out from other alternatives, consistently ranking at or near the top across most TFIs indicators. Conversely, Indonesia ranks at the bottom, having the lowest or near-lowest TFIs indicator values.

5. Sensitivity analysis

To emphasize the robustness of the study, different sensitivity analysis scenarios can be performed, and the results compared with the initial findings. This study applies eight weighting methods, along with mean weights, for sensitivity analysis.

MABAC	MEREC	WENSLO	ENTROPY	LOPCOW	CVM	CRITIC	ANGLE	GINI	mean
Brazil	-0.041	-0.037	-0.057	-0.003	-0.008	-0.054	-0.050	-0.054	-0.038
China	0.068	0.015	0.025	0.087	0.090	0.104	0.074	0.076	0.067
Indonesia	-0.325	-0.292	-0.294	-0.290	-0.282	-0.201	-0.290	-0.288	-0.283
India	-0.034	-0.031	-0.027	-0.089	-0.085	-0.064	-0.038	-0.035	-0.051
Mexico	0.196	0.275	0.255	0.140	0.136	0.090	0.186	0.182	0.183
Russia	0.252	0.232	0.237	0.250	0.251	0.245	0.253	0.252	0.246
Türkiye	0.133	0.080	0.108	0.157	0.151	0.139	0.117	0.120	0.125
Brazil	6	6	6	5	5	5	6	6	5
China	4	4	4	4	4	3	4	4	4
Indonesia	7	7	7	7	7	7	7	7	7
India	5	5	5	6	6	6	5	5	6
Mexico	2	1	1	3	3	4	2	2	2
Russia	1	2	2	1	1	1	1	1	1
Türkiye	3	3	3	2	2	2	3	3	3

Table 6: *MABAC method findings based on the different weighting methods.*

In the MABAC method (see Table 6), which ranks the alternatives based on the determined approximation areas, different rankings were obtained depending on the criteria weights used.

Russia consistently ranks first, except when using the WENSLO and ENTROPY weights, while Indonesia consistently ranks last. In other words, minimal change was observed compared to the rankings found using the ANGLE weights as shown in Figure 2.

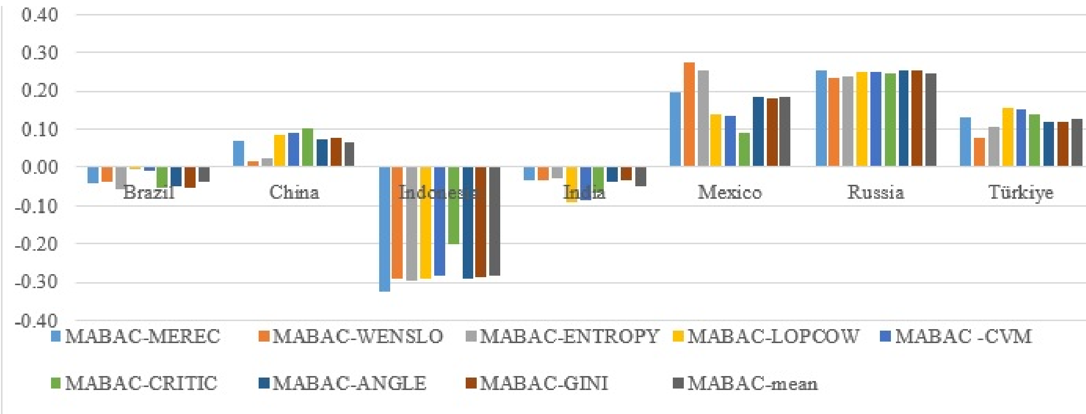

Figure 2: *MABAC method rankings.*

The rankings and sensitivity analysis of alternatives can be found based on previously calculated criteria weights not only for the MABAC method but also for the ARTASI method. The findings of the ARTASI method, using different methods, are presented in Table 7.

ARTASI	MEREC	WENSLO	ENTROPY	LOPCOW	CVM	CRITIC	ANGLE	GINI	mean
Brazil	1.635	1.597	1.589	1.701	1.694	1.656	1.622	1.619	1.639
China	1.690	1.600	1.615	1.760	1.757	1.750	1.681	1.682	1.692
Indonesia	1.424	1.393	1.401	1.495	1.496	1.538	1.440	1.441	1.453
India	1.654	1.596	1.608	1.686	1.681	1.677	1.635	1.636	1.647
Mexico	1.817	1.832	1.824	1.815	1.809	1.770	1.802	1.799	1.808
Russia	1.831	1.780	1.789	1.866	1.863	1.846	1.821	1.821	1.827
Türkiye	1.752	1.676	1.700	1.796	1.789	1.767	1.731	1.732	1.743
Brazil	6	5	6	5	5	6	6	6	6
China	4	4	4	4	4	4	4	4	4
Indonesia	7	7	7	7	7	7	7	7	7
India	5	6	5	6	6	5	5	5	5
Mexico	2	1	1	2	2	2	2	2	2
Russia	1	2	2	1	1	1	1	1	1
Türkiye	3	3	3	3	3	3	3	3	3
MEREC	1								
WENSLO	0.929	1							
ENTROPY	0.964	0.964	1						
LOPCOW	0.964	0.964	0.929	1					
CVM	0.964	0.964	0.929	1	1				
CRITIC	1	0.929	0.964	0.964	0.964	1			
ANGLE	1	0.929	0.964	0.964	0.964	1	1		
GINI	1	0.929	0.964	0.964	0.964	1	1	1	
mean	1	0.929	0.964	0.964	0.964	1	1	1	1

Table 7: *ARTASI method findings based on the different weighting methods.*

The ARTASI method, which ranks alternatives within specified limits, produces identical rankings when using MEREC, CRITIC, ANGLE, GINI and mean weights. The WENSLO, ENTROPY, LOPCOW, and CVM methods show strong positive correlations, with the lowest correlation at 0.929.

As in the MABAC findings, Russia ranks first, except when using WENSLO and ENTROPY weights, and Indonesia consistently ranks last. Given the narrow evaluation range (0-2), the ARTASI method produces values that are close but sortable. When all rankings obtained using MABAC and ARTASI methods were evaluated, identical rankings were produced across different methods in both approaches except for WENSLO and CRITIC.

6. Discussion and managerial implications

Since no prior studies have addressed TFI calculations, this study aimed to make a comparison through a unique application by diversifying and increasing the number of alternatives. TFIs data from 38 OECD members were used to make comparisons, with calculations conducted on a more comprehensive decision matrix using the same methods, as shown in Table 8.

	MEREC	WENSLO	ENTROPY	LOPCOW	CVM	CRITIC	ANGLE	GINI	mean
w1	0.057	0.042	0.042	0.091	0.089	0.076	0.068	0.071	0.067
w2	0.055	0.055	0.06	0.075	0.072	0.1	0.081	0.083	0.073
w3	0.091	0.086	0.12	0.085	0.082	0.113	0.112	0.099	0.098
w4	0.062	0.109	0.096	0.066	0.064	0.096	0.103	0.111	0.088
w5	0.033	0.023	0.024	0.074	0.071	0.133	0.052	0.054	0.058
w6	0.099	0.091	0.113	0.093	0.091	0.093	0.108	0.104	0.099
w7	0.074	0.051	0.053	0.100	0.099	0.101	0.076	0.079	0.079
w8	0.036	0.033	0.031	0.070	0.067	0.09	0.059	0.063	0.056
w9	0.101	0.141	0.149	0.082	0.079	0.077	0.127	0.135	0.111
w10	0.317	0.343	0.288	0.127	0.137	0.06	0.166	0.161	0.200
w11	0.074	0.026	0.023	0.136	0.151	0.062	0.05	0.04	0.070
MEREC	1								
WENSLO	0.958	1							
ENTROPY	0.929	0.978	1						
LOPCOW	0.601	0.386	0.334	1					
CVM	0.590	0.380	0.318	0.996	1				
CRITIC	-0.539	-0.477	-0.409	-0.680	-0.688	1			
ANGLE	0.825	0.915	0.971	0.190	0.168	-0.333	1		
GINI	0.766	0.895	0.939	0.073	0.052	-0.313	0.981	1	
mean	0.977	0.985	0.984	0.479	0.465	-0.479	0.923	0.878	1
E7-OECD	0.403	0.798	0.799	0.112	0.148	-0.063	0.756	0.735	0.746

Table 8: *Comparisons for a different data set based on the same weighting methods.*

Table 8 shows the TFIs calculated using the same methods as before, this time for OECD countries. Notably, the "w10-External border agency co-operation" indicator was found to be more important than the others, but this time even more important (0.200). Another noteworthy is "w9-Internal border agency co-operation" indicator, which was previously the third most important, emerged as the second most important criterion (0.111) this time. Again, very high correlations were found between LOPCOW-CVM (0.996), ANGLE-GINI (0.981), and ENTROPY-WENSLO (0.978). Furthermore, when analyzing the calculations/weightings for E7 (seven countries) and OECD (38 countries) in the last row of Table 8, it turns out that the weights before and after analysis are highly correlated in the ENTROPY (0.799) and WENSLO (0.798) methods. The ANGLE and GINI methods again emerged as the most appropriate methods when examining the distances of eight methods to the mean using multidimensional scaling analysis for OECD countries. Additionally, the ANGEL-based MABAC and ARTASI methods were applied to OECD countries, Korea, Netherlands, Sweden, Japan, and Norway ranking in the top five as trade-facilitating countries. On the other hand, the study also offers important theoretical and managerial implications, contributing to the literature.

Trade facilitation offers consumers with a wider range of products and services while providing employment and greater profit opportunities for producers. Facilitating factors can attract international investments, leading to new job opportunities for investors through more predictable trade practices. Trade facilitation helps countries to transfer technology, thus increasing their competitiveness. Trade facilitation increases cooperation between countries, promoting more peaceful solutions. It also increases overall social welfare by indirectly contributing to countries in areas such as infrastructure, health and social development. The benefits of trade facilitation for countries include encouraging economic growth, increasing productivity and efficiency, benefiting both producers and consumers, increasing investments, indirectly providing technology transfer, increasing international cooperation and communication, ensuring sustainable development and long-term welfare.

Policymakers should take measures to trade facilitation, develop long-term strategic plans in this direction, and prioritize external-internal border agency co-operation, which this study identifies as particularly important. At the same time, countries should enhance trade facilitation by increasing the use of digital systems and automation to make customs procedures and transactions faster and more efficient, making customs duties and transactions more transparent and predictable, reducing trade barriers and tariffs, developing logistics and digital infrastructure, guiding companies in foreign trade, being transparent in governance, increasing human resources and entrepreneurship, supporting technology and innovation, and implementing a sustainable trade policy by effectively using international diplomacy. Policymakers need to prioritize reforms, such as reducing export costs, facilitate customs clearance, and support small and medium-sized enterprises, even though they are often costly and difficult to implement. Ultimately, it is recommended to prioritize policy tools and reform measures to accelerate export growth and economic development through trade facilitation.

7. Concluding remarks and future perspectives

The globalizing world is becoming more complex, prompting countries to introduce new measures in their trade with other countries to protect their domestic producers. On the other hand, countries that aim to support growth and development through increased trade are making efforts to eliminate trade barriers. This study analyses trade facilitation, using different methodologies, to develop a comprehensive understanding of trade facilitation for the E7 and comparable OECD countries. It also examines the effectiveness and usability of different methods, illustrated with diverse examples. The empirical results show that: 1. Among trade facilitation indicators, external border agency cooperation is the most significant factor 2. In the MABAC and ARTASI calculations, the E7 countries that are leading in trade facilitations, in order, are Russia, Mexico, Turkey, China, India, Brazil and Indonesia. 3. Russia ranks first among the E7 countries in terms of trade facilitation. However, since Russia's full-scale invasion of Ukraine, its international trade has undergone significant changes in both overall size and geographic composition of exports and imports. A coalition of Western countries has imposed comprehensive economic sanctions, which include restrictions on access to export markets, imports, logistics and financing. Additionally, measuers such as freezing the assets of the Russia's Central Bank, blocking access to foreign currency reserves, excluding Russia from SWIFT, and targeting certain individuals, have been implemented. 4. The study demonstrated that the ANGLE-MABAC model can be effectively used in future studies involving similar decision-making problem.

Trade facilitation contributes the volume of foreign trade and enhances the economic growth of countries, indirectly supporting job creation, reducing unemployment, increasing incomes, and improving overall economic welfare. Trade facilitation plays a positive role in eliminating competitive barriers in the trade of goods between countries, simplifying customs procedures, and encouraging companies to become more efficient and develop innovative solutions. This article could be improved in the following aspects. First, the proposed methodology could be applied to a comparison across all countries, globally. Additionally, current and newly introduced methods could be compared to assess the proposed methodology's robustness. Another limitation to note is the use of secondary data; instead, a study could involve subjective methods by directly obtaining opinions from experts on the subject, but in this case, it would have been quite limiting both in terms of reaching the right experts and in terms of time and resource constraints.

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TOPSIS-based framework for evaluating employee cybersecurity risk

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Abstract. This paper presents the development and initial validation of a TOPSIS-based framework for evaluating employee cybersecurity risk (TFEECR). The framework offers a structured and objective approach to assessing multiple dimensions of employee behavior and attitudes toward cybersecurity. It integrates the Technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS) with Saaty's criteria weighting, enabling a comprehensive evaluation across five criteria: knowledge, compliance, risky behaviors, attitudes, and training. The framework is validated using simulated data, and it showed robustness in differentiating between high- and low-risk employees. Such results are useful for organizations that want to implement targeted interventions. A sensitivity analysis highlights the framework's adaptability to various settings. This conceptual framework lays the groundwork for future empirical validation and practical application in enhancing organizational cybersecurity.

Keywords: cybersecurity risk assessment, employee behavior, MCDM, risk management, TOPSIS framework

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1. Introduction

Managing cybersecurity risks is becoming more important for organizations to operate successfully in the digital era. Cyber threats are becoming more sophisticated, meaning that companies need to continuously monitor and develop their security posture and a resilient cybersecurity framework. Since employees are integral to these frameworks, it is mandatory to assess and reduce the risks related to human behavior. Employees who lack knowledge and skillsets are seen as a susceptible threat vector for cyber attacks, making them vulnerable targets for attackers [7]. The increased vulnerability of organizations during extreme events, as observed in 2020-21, underscores the importance of robust knowledge risk strategies, including proper knowledge identification and guidelines for online behavior, to mitigate the heightened employee-related cybersecurity risks [1, 14], ever more important in ongoing digitalization processes.

Conventional risk assessment methods often overlook the human factor or do not consider all variables or different evaluation criteria related to human behavior. Even if they do consider them, evaluations are often complicated by the subjective nature of human reporting and the complexity of integrating qualitative and quantitative data in comprehensive assessments. The evaluation of employee-related cybersecurity risks is a complex process that considers a number of variables [4, 11, 29]. The evaluation is usually conducted through data collection, measuring scale, and analysis [12]. However, using only subjective measurements from self-assessment questionnaires adds a subjective element that may be harmful to the evaluation procedure [12].

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Moreover, the importance of employing multi-criteria optimization methods in cybersecurity and information security tasks is emphasized to overcome these limitations and ensure accurate decision-making [13]. That points to a gap and a need for a systematic approach to evaluating multiple criteria that affect an employee's cybersecurity risk profile. Therefore, developing a more objective evaluation process of employee-related cybersecurity risks is a promising research direction.

The goal of this paper is to present the framework for evaluating employee-related cybersecurity risk, which offers a systematic way to evaluate and rank employees according to their degrees of cybersecurity risk, which can be further tested in future research. In order to achieve a comprehensive approach, multiple factors – knowledge, compliance, risky behaviors, attitudes, and training – will be considered, in combination with a Multi-Criteria Decision-Making (MCDM) technique, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). This combination creates a TOPSIS-based Framework for Evaluating Employee Cybersecurity Risk, or TFEECR for short. In such a way, the aim is to provide a conceptual framework that can provide organizations with a holistic, objective, and scalable approach to improve their entire cybersecurity posture and direct future empirical research for further validation.

This paper advances the cybersecurity risk assessment literature by presenting a TOPSIS-based framework for evaluating employee-related cybersecurity risks. The proposed framework integrates multiple criteria, and enables the assessment of employees based on their cybersecurity risk levels, which allows organizations to implement targeted measures to reduce these risks. This systematic approach offers a scalable and customizable framework that can be tailored to various organizational needs, thereby enhancing the overall cybersecurity posture. The research contributes to filling the noted gap in the literature regarding the use of quantitative methods in cybersecurity risk evaluations and provides a basis for subsequent empirical investigations and practical implementation in organizational contexts.

The subsequent sections of the paper are structured as follows: the second section presents an overview of the methodologies employed, encompassing the combination of the TOPSIS framework with Saaty's criteria weighting. The third section delineates the analysis and outcomes of the framework applied to simulated employee data. The fourth section examines the ramifications of the findings and their practical utilization. The conclusion encapsulates the primary contributions and proposes avenues for future research.

2. Methods

Cybersecurity risk management involves complex decision-making processes, which require considering multiple criteria. So, simpler, single-criterion models cannot capture the versatile nature of cybersecurity threats and related human behavior, which involves various desirable and undesirable traits and behaviors that can be identified. According to Khoroshko et al. [13], using multi-criteria optimization methods in cybersecurity and information security tasks can overcome these limitations, leading to the enhanced correctness of decision-making. The importance of quantitative methods in developing practical applications and filling in research gaps is also stressed by the bibliometric study of cybersecurity threats [16]. Quantitative methods enable continuous improvement and self-assessment in companies, which allows them to improve their security posture by making informed and cost-effective decisions [8].

According to Saaty's criteria for MCDM methods [21], TOPSIS is particularly advantageous when the decision problem involves ranking alternatives based on multiple conflicting criteria and there is a need for simplicity and computational efficiency. Moreover, TOPSIS is highly versatile when it comes to data types and scenarios. It works best with numeric data and is suited to handling criteria with different metrics and scales, where it handles variability in the data quite effectively. Since TOPSIS focuses on identifying the best alternative based on the relative closeness to an ideal solution, it makes it suitable and "intuitive" for HR decisions where scoring is relative to an "ideal employee".

While preferred for its simplicity and versatility, TOPSIS can be outperformed by other methods in some scenarios. For example, PROMETHEE is a better choice in any situation where nuanced or non-linear human preferences, non-compensatory trade-offs, or complex outlier handling are needed because it handles them with flexible preference functions and allows for visual interpretability. In some cases, TOPSIS and PROMETHEE can both be suitable and produce similar results, although the output will be sensitive to the criteria used [19]. PROMETHEE is deemed more appropriate for partial ranking and decision-making in situations where, for example, environmental issues or sustainability considerations require more tailored judgments [23]. However, it can become complicated in terms of requiring expert input and adjusting the preference threshold, whereas TOPSIS stands out for its simplicity and uses distance-based measures to rank alternatives, making it more suitable when a clear numerical assessment is possible. Furthermore, ELECTRE is more complex than TOPSIS and is particularly suitable for handling ambiguous, non-compensatory criteria. However, it is used for exclusion-based ranking rather than optimality-based ranking like TOPSIS. VIKOR is similar to TOPSIS in its simplicity but focuses more on maximizing group utility and minimizing regret. However, VIKOR is more complex than TOPSIS and will not be a first choice in scenarios where straightforward ranking is needed. Another commonly applied method, SMART, is a very straightforward, simple, and intuitive method where criteria are weighted, and alternatives are scored based on a weighted sum of criteria values. However, the downside is that SMART assumes linear trade-offs between criteria and does not account for the distances between alternatives and ideal points, as TOPSIS does. In addition, AHP is one of the most frequently applied methods, which allows pairwise comparisons to determine the relative importance of each criterion and provides a structured framework for decision-making. Still, it cannot deal with conflicting criteria. Considering the goal – to develop a framework for evaluating employee-related cybersecurity risk – where numerical input is expected, with different metrics and scales, along with the conflicting criteria, TOPSIS is the simplest method that is effective and versatile enough to handle such demands.

TOPSIS is one of the MCDM methods, introduced in 1981 [10] and has been widely applied since. It has been successfully applied to assess cybersecurity risks in multiple sectors [30, 31, 33]. The method has proved to be useful in complex, practical domains [3, 19, 23].

TOPSIS is primarily developed to identify the best alternative among a set of options based on their relative closeness to an ideal solution [10]. That is achieved by first determining the best and worst solutions in the data and then ranking the alternatives based on their relative closeness to these ideal solutions. But, in doing so, it also ranks all the alternatives, which broadens its possible application. It fully utilizes attribute information, offers a cardinal ranking of alternatives, and does not require the independence of attribute preferences [3]. The technique can handle multiple, often conflicting, criteria and provide actionable rankings. Additionally, it supports the integration of both qualitative and quantitative data, allowing for a comprehensive evaluation. The technique uses a simple and effective approach to data normalization, distance calculation, and ranking, which enhances the ease of implementation and encourages its wide use [3].

In the context of employee-related cybersecurity risk, an important TOPSIS characteristic is that it integrates both beneficial and non-beneficial criteria, thus allowing the evaluation to be based on desirable and non-desirable attributes simultaneously. For example, while knowledge is a beneficial criterion and desirable behavior, risky behavior is not a desirable behavior and represents a non-beneficial criterion.

In addition, some evaluations can be made using validated questionnaires and tests, but they can be measured on different scales and result in different score intervals. On the other hand, some evaluations, such as observed behavior, can be conveyed as qualitative information and then coded. Moreover, the values can differ in measurement units, where the knowledge can be a test result measured in points, and training can be measured in hours. Normalization, which is the first step of this method, allows for comparison of such data.

These characteristics show the technique's flexibility and allow for application to various (complex) settings. It can be argued that this technique is particularly appropriate for assessing employee-related cybersecurity risks because it upholds a combination of possibly conflicting criteria, both quantitative and qualitative data, and computationally efficient calculations. The method application unfolds as follows.

To apply the TOPSIS method effectively, it is important to determine the weights that reflect the relative importance of each criterion in assessing employee-related cybersecurity risks. Saaty's criteria for MCDM [21] highlight the importance of structured frameworks for prioritizing criteria and assigning weights. One commonly used method for determining these weights is Saaty's pairwise comparison, which is used at the beginning of the Analytic Hierarchy Process (AHP). It involves comparing each criterion against others to establish their relative importance verbally and then translating them into numerical values using a scale of one to nine [22]. These comparisons are organized into a matrix, which is then normalized, and the eigenvector is calculated to determine the weights for each criterion. This process ensures that the criteria weights reflect their significance in evaluating employee-related cybersecurity risks while maintaining consistency in judgments.

While this consideration clearly shows that not all criteria are equally important in an employee profile, TOPSIS's classical approach to weighting alternatives stems from the variation in the data rather than utilizing the practical meaning of the criteria. However, borrowing from the Saaty criteria weighting can help to deal with the importance of criteria. At the same time, AHP does not support the non-beneficial criteria in the ranking, so it is not appropriate for the entire process. Nevertheless, there is no obstacle in combining the weights derived by pairwise comparison on the Saaty scale and TOPSIS, as has been shown by previous research, where this approach of using Saaty criteria weighting to determine criterion weights and TOPSIS to rank alternatives based on closeness to the ideal solution. It is referred to as the AHP-TOPSIS method or the AHP-TOPSIS method combination [23, 24].

2.1. Goal definition

The goal is to define and test the framework to assess employee-related cybersecurity risks within an organization using the TOPSIS method. By systematically evaluating individual employees based on multiple relevant criteria, the framework aims to identify best-behaving and high-risk individuals to enable the implementation of targeted interventions to mitigate these risks. This proactive approach helps strengthen the organization's overall security posture, ensuring that all employees contribute to a safer digital environment.

2.2. Criteria identification

Evaluating employee-related cybersecurity risks requires a comprehensive approach that considers multiple dimensions of knowledge and behavior. The assessment of employees as cybersecurity risk is proposed to be based on several key traits in different combinations, such as knowledge, attitudes, skills, and behavioral intentions [29].

Knowledge of cybersecurity threats and best practices is foundational, as it directly influences employees' ability to recognize and respond to potential risks [17]. It involves employee's understanding of fundamental security principles, protocols, and practices. This includes awareness of common cyber threats and knowledge of protective measures like strong passwords, multi-factor authentication, and encryption. Studies show that security knowledge positively impacts both security compliance and participation, with threat appraisal (severity and susceptibility) further strengthening these effects [6]. The role of knowledge is further underscored by the findings that employees familiar with their company's cybersecurity policies are better equipped to handle related tasks, suggesting that organizational environments fostering cybersecurity awareness positively impact employee behavior [28]. Yet, data breaches persist due to insider actions, highlighting the need for a more nuanced understanding of security knowledge breadth, depth, and finesse [26].

Compliance with cybersecurity policies is another critical factor that involves the adherence to established security policies, procedures, and regulatory requirements within an organization. It consists of following guidelines related to data protection, password management, access controls, and incident reporting [17]. Compliance ensures that all members of the organization are aligned with best practices and legal mandates, reducing vulnerabilities and preventing security breaches. For instance, cybersecurity awareness programs that enhance knowledge and attitudes toward compliance can lead to better adherence to information security policies and protective behaviors [27]. The assessment of employees as cybersecurity risk is proposed through the components of compliance and participation in cybersecurity behavior [6]. Bélanger et al. [4] suggested that organizations implement holistic employee compliance programs that include repeated exposure to security and privacy news to improve employees' information protective behaviors.

Attitudes toward cybersecurity encompass employees' perceptions, beliefs, and mindsets regarding the importance and impact of security measures [5, 17]. Positive attitudes, such as recognizing the value of security protocols and feeling responsible for protecting sensitive information, contribute to a more proactive and vigilant security culture. Employees' attitudes toward cybersecurity risks were found to have a significant impact on their engagement in risky online behaviors [9]. Cultivating a positive security mindset through education and communication is vital for fostering a culture of security awareness and accountability.

Training is the process of educating employees about cybersecurity best practices, potential threats, and appropriate responses to security incidents. Regular, comprehensive training sessions help reinforce good security habits, update employees on emerging threats, and ensure that security protocols are understood and followed. Effective training methods, such as text-based and game-based formats, have been shown to significantly influence employees' attitudes and behaviors toward cybersecurity [2]. Additionally, periodic training and awareness programs are recommended to counteract the inherent risks in cyberspace and ensure employees are prepared for potential cyber threats [18, 28]. The training is suggested to include characteristics of cyberspace risk and specific guidelines for employees to enhance their awareness and preparedness in dealing with potential cyber threats [18]. Nevertheless, research by Renaud and Goucher [20] suggests that employees, even those with the required knowledge, sometimes compromise the security of an organization's systems and information by behaving insecurely, emphasizing the need to understand how employees are expected to assist in keeping the organization's information secure.

Risky behaviors, such as neglecting password management or mishandling information, may be a result of insufficient knowledge and poor attitudes toward cybersecurity and increase the likelihood of security breaches. Examples include using weak or reused passwords, clicking on suspicious links, downloading unverified software, neglecting to update systems and software, and a lack of active cyber defense measures while working from home [14, 25]. Identifying and mitigating these behaviors is crucial, as they can create significant security gaps that cybercriminals may exploit [11]. It is suggested that by promoting awareness and encouraging safer practices, organizations can reduce the risk posed by such behaviors and strengthen their overall security defenses.

The relevant criteria for evaluating employee-related cybersecurity risks are encompassed by the umbrella terms of knowledge, compliance, risky behaviors, attitudes, and training (Table 1). These criteria capture the multifaceted nature of employee cybersecurity behavior outlined in Ali et al.'s [1] systematic review of information security policy compliance. By considering these diverse criteria, the framework provides a comprehensive assessment of an employee's cybersecurity risk profile.

Objective	**Criteria**	**Description**	**Common measures**	**References**
Minimize risky behaviors	Risky behavior	actions that increase vulnerability to cyber threats	questionnaire, Behavioral-Cognitive Internet Security Questionnaire (BCISQ)	[1, 14, 20, 25]
Increase cybersecurity knowledge	Knowledge	understanding of fundamental cybersecurity principles and threats	questionnaire, Human Aspects of Information Security Questionnaire (HAIS-Q)	[1, 6, 17, 29]
Enhance compliance with policies	Compliance	adherence to established security policies and procedures	questionnaire, Human Aspects of Information Security Questionnaire (HAIS-Q)	[1, 4, 6, 17, 27, 29]
Improve attitudes toward security	Attitudes	perceptions and mindsets toward cybersecurity policies and measures	questionnaire, Security Attitude Inventory (SA-13), Human Aspects of Information Security Questionnaire (HAIS-Q)	[1, 5, 17, 29]
Promote sufficient training participation	Training	participation in cybersecurity education programs	attended or not; level of training; hours of training	[1, 2, 4, 6, 27, 28]

Table 1: *Criteria for employees' overall cybersecurity behavior.*

So, the approach to the pairwise comparison is based on the deduction from the theoretical framework (Table 2), and more detailed reasoning for the comparisons can be found in the Supplementary file. The table was constructed using the Saaty scale. Risky behaviors are identified as the most critical factor in cybersecurity risks because they can directly lead to incidents like data breaches and malware infections, regardless of an employee's knowledge level [4, 20]. Whereas risky behavior may be understood as a habitual version of non-compliance due to bad habits, or carelessness, compliance refers to behaving in line with the security rules. However, being compliant does not automatically mean that risky behaviors are entirely avoided, so they will not have equal weight, with Risky behavior being slightly more important. Compliance ensures adherence to security protocols [4], applying the gained Knowledge so they are interconnected and evaluated as equally important. Attitudes toward cybersecurity influence the overall security culture and employees' motivation to follow best practices. Still, without Knowledge, Compliance, and the absence of Risky Behaviors, they cannot lead to the desired behavior. At the same time, training, though essential, depends on the proper integration of knowledge, compliance, and attitudes to be effective [9]. Training per se is a requirement, but it is not sufficient without the other criteria and thus least relevant.

Criteria	Risky Behaviors	Knowledge	Compliance	Attitudes	Training
Risky Behaviors	1	2	2	3	5
Knowledge	1/2	1	1	2	4
Compliance	1/2	1	1	2	3
Attitudes	1/3	1/2	1/2	1	2
Training	1/5	1/3	1/3	1/2	1

Table 2: *Pairwise comparison.*

This resulted in the following priorities, that will further be used as weights: $w_{RB} = 0.38$, $w_K = 0.22$, $w_C = 0.21$, $w_A = 0.12$, and $w_T = 0.07$. The consistency ratio of these criteria is 0.0055, which indicates that the pairwise comparisons made using the Saaty scale are highly consistent, thus ensuring the reliability of the derived weights and minimizing the risk of random or biased judgments in the comparison process.

2.3. Simulated data

The data is simulated and involves generating synthetic employee data across five cybersecurity risk-related criteria: Risky Behavior, Knowledge, Compliance, Attitudes, and Training. Target means, standard deviations, and correlations are derived from real-world studies such as the Behavioral-Cognitive Internet Security Questionnaire (BCISQ) [25], the Human Aspects of Information Security Questionnaire (HAIS-Q) [15, 17], and the Security Attitude Inventory (SA-13) [5]. These data are used to generate distributions, which are then refined iteratively to match the statistical properties of the target dataset. The questions, metrics from previous surveys, data generation, and procedure are available in the Supplementary files.

Statistics	Knowledge	Compliance	Risky behaviors	Attitudes	Training
Minimum	65	66	3.984	31	0
Q1	76.75	79	6.332	40.75	0
Median	81.5	85	7.42	46	1.474
Mean	81.34	84.9	7.41	45.83	1.538
Q3	86	90.25	8.643	50	2.276
Maximum	97	103	11.105	60	6.959

Table 3: *Insight into the simulated data.*

The dataset's (Table 3) diversity in metrics, scales, ranges, and variation highlights the advantages of using TOPSIS. Or, the other way around, the method's ability to handle different variations, ranges, metrics, and scales, as well as both benefit and cost criteria, confirms it is particularly well-suited for the analysis of such data. While using simulated data in multi-criteria decision-making (MCDM) methods, such as TOPSIS, is not typical, it is not an uncommon approach to assess a method's applicability or to compare methods [32]. Simulated data enables a conceptual demonstration of the framework before it undergoes empirical testing. This approach allows for the evaluation of its applicability, potential (dis)advantages, and requirements, offering valuable insights before exposing respondents to extensive questionnaires.

2.4. Analysis and results

After the goal definition, criteria are determined, and data is generated, the next step is to construct a decision matrix. Each element in the matrix represents the performance score of an alternative with respect to a criterion. Suppose there are m alternatives and n criteria; the decision matrix D can be represented as: $D = [x_{ij}]$ for $i = 1, \dots, m$, and $j = 1, \dots, n$, where x_{ij} is the score of the i-th alternative with respect to the j-th criterion. The next step is normalization. In TOPSIS, the normalization method typically uses vector normalization, which transforms the various criteria dimensions into non-dimensional criteria, allowing for comparisons across different scales. The normalized value r_{ij} is calculated as:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \tag{1}$$

which ensures that all criteria have equal importance in subsequent calculations.

The box plots of the normalized data indicate that the distribution of values across the criteria has been effectively transformed to a comparable scale (available in the Supplementary file, Figure 3). The variability within each criterion is evident from the length of the boxes and the range of the whiskers, highlighting the diversity in employee scores across the different cybersecurity-related attributes. Next, each criterion is assigned a weight w_i reflecting its relative importance. The weighted normalized value v_{ij} is computed as:

$$v_{ij} = w_j \cdot r_{ij}, \tag{2}$$

where $\sum_{j=1}^{n} w_j = 1$. The results of the implementation of weights can be observed in Figure 1.

Figure 1: *Box plots of the weighted normalized values.*

This is followed by the calculation of the positive ideal solution (A^*) and the negative ideal solution (A^-). A^* and A^- are determined based on the weighted normalized decision matrix. For beneficial criteria (higher values are better), and are defined as:

$$A^* = \{v_1^*, v_2^*, \dots, v_n^*\} = \left\{\max_i v_{ij}\right\} \tag{3}$$

$$A^- = \{v_1^-, v_2^-, \dots, v_n^-\} = \left\{\min_i v_{ij}\right\} \tag{4}$$

For non-beneficial criteria (lower values are better), the values are reversed:

$$A^* = \{v_1^*, v_2^*, \dots, v_n^*\} = \left\{\min_i v_{ij}\right\} \tag{5}$$

$$A^- = \{v_1^-, v_2^-, \dots, v_n^-\} = \left\{\max_i v_{ij}\right\} \tag{6}$$

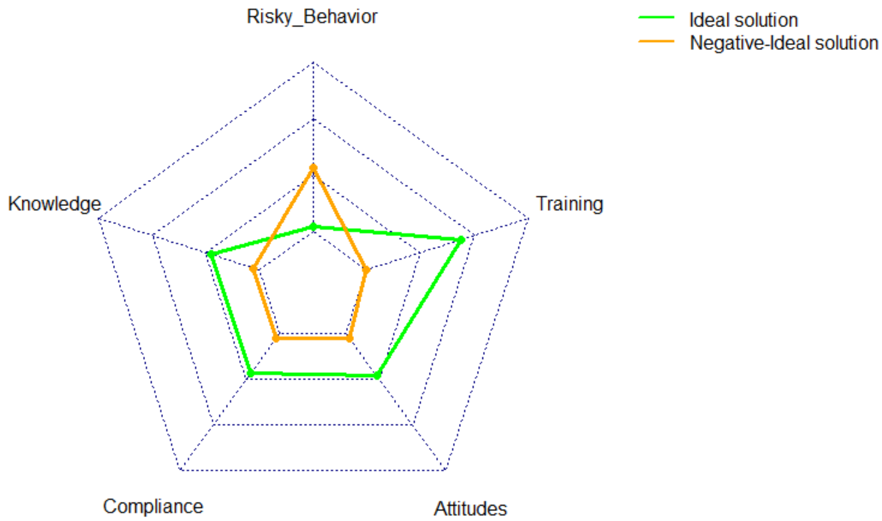

Figure 2: *Ideal vs negative ideal solution.*

The ideal and negative ideal solutions in TOPSIS are determined by evaluating the criteria based on their benefit or cost nature (Figure 2). The difference between the ideal and negative ideal solutions depends on the data's distribution and the criteria values. If the criteria values are relatively close to each other after normalization and weighting, the ideal and negative ideal solutions might not show a large difference.

Risky behaviors are identified as the most critical factor, and they obtained the highest weight. As they directly undermine cybersecurity efforts, risky behaviors are considered non-beneficial criteria in the TOPSIS framework. This means that lower values of risky behaviors are more desirable, and thus, the goal is to minimize these behaviors to reduce overall cybersecurity risk. This ensures that the ideal solution represents the most favorable conditions, and the negative ideal solution represents the least favorable conditions for each criterion.

The separation measures, calculated in the next step, are the Euclidean distances of each alternative from the positive and negative ideal solution. The separation from A^* is S_i^* and from A^- is S_i^-, and are calculated as:

$$s_i^* = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^*)^2} \tag{7}$$

$$s_i^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2} \tag{8}$$

These separation distances are further used to compute the relative closeness of each employee to the ideal solution. The relative closeness of each alternative to the ideal solution C_i^* is computed as:

$$C_i^* = \frac{s_i^-}{s_i^* + s_i^-} \tag{9}$$

where $0 \leq C_i^* \leq 1$. The alternative with a C_i^* value closer to 1 is preferred, as it is nearer to the positive and farther from the negative ideal solution. The last step is to rank employees based on their relative closeness scores, C^*. The alternative with the highest C_i^* value is considered the best choice, or in this case, the highest rank. Such ranking allows the identification of the best-performing and high-risk individuals and recommends interventions.

Empl.	**Risky Behavior**	**Knowledge**	**Compliance**	**Attitudes**	**Training**	**C***	**Rank**
Emp_96	0.034	0.035	0.044	0.035	0.063	0.678	1
Emp_56	0.056	0.045	0.040	0.036	0.087	0.668	2
Emp_16	0.054	0.036	0.039	0.036	0.082	0.653	3
Emp_41	0.027	0.021	0.024	0.024	0.076	0.650	4
Emp_51	0.047	0.037	0.042	0.046	0.054	0.604	5
...							
Emp_35	0.026	0.024	0.020	0.020	0.000	0.297	96
Emp_14	0.016	0.010	0.012	0.012	0.006	0.294	97
Emp_54	0.014	0.012	0.009	0.008	0.000	0.286	98
Emp_39	0.027	0.020	0.022	0.020	0.000	0.282	99
Emp_49	0.017	0.011	0.011	0.013	0.000	0.276	100

Table 4: *Bottom and top ranked employees.*

The rankings derived from the TOPSIS method reveal insights into employee cybersecurity risks (Table 4). The top-ranked employees exhibit high compliance, knowledge, attitudes, and training scores, coupled with low risky behavior scores, underscoring their adherence to security protocols and proactive security measures. Conversely, the bottom-ranked employees demonstrate significant gaps in these areas, highlighting the need for targeted awareness programs, training, and reinforcement to mitigate their potential risks. These findings emphasize the effectiveness of the TOPSIS-based framework in pinpointing specific areas for improvement and guiding strategic interventions to enhance the organization's cybersecurity resilience.

2.5. Evaluation

Several approaches were applied to evaluate the proposed framework. Through the examination of the separation distances from ideal and negative-ideal solutions, analysis of relative closeness scores and criteria, and examining the sensitivity of the framework to changes in criteria weights, the aim is to demonstrate the framework's ability to differentiate between employees and validate its robustness.

The variations in the distances from both the ideal and negative ideal solutions across employees indicate that the method captures differences in their performance (Figure 6 in the Supplementary file). If the lines were flat or showed little variability, it would suggest the method is not distinguishing well between the employees. In addition, the plot shows that some employees are closer to the ideal solution while others are closer to the negative ideal solution. This range of performance reflects the method's ability to differentiate between high and low performers. While the green and orange lines come very close together for some employees, which indicates the method is struggling to differentiate effectively in these cases. However, the plot shows sufficient separation overall, so it can be concluded that the framework is differentiating employees fairly well.

The relationship between the results, that is, relative closeness scores, to initial employees' values per each criterion (Figures 8-10 in the Supplementary file) reveals that values for any criterion observed alone cannot lead to the prediction of the final scores. That can be observed in similar spreads of C* scores, where no criterion has a very strong correlation (either positive or negative) compared to the others. That additionally emphasizes the holistic and intertwined evaluation based on the beneficial and non-beneficial criteria applied to the data with high variability.

However, criteria still have a meaningful impact on C*, meaning that the overall framework is sensitive enough to differentiate between employees based on all the criteria. The points in the scatter plot are sufficiently spread out, which indicates that the method successfully differentiates between employees.

Sensitivity analysis in TOPSIS typically involves examining how changes in input data, such as criteria weights, affect the final assessment outcomes. Additional analysis of changes in the weights was conducted to examine the effect of varying each criterion's weight across a range of values (from 0.01 to 0.45, increasing by 0.01) while proportionally adjusting the other criteria weights. This allows for the assessment of the sensitivity of final rankings to changes in criteria weights, which are essential parts of the framework.

The systematically varied criteria weights were used to calculate the employee rankings, and then they were compared with the original rankings using Spearman's rank correlation coefficient (the complete analysis is available in the Supplementary file). A threshold of 0.7 for the correlation coefficient was selected based on Chaddock's scale, denoting the boundary between moderate and strong correlations, and further used to identify criteria weight sets that produced closely aligned rankings. For these selected weight sets, the minimum and maximum values of each criterion were determined, representing the range of weights that resulted in similar rankings. These wide ranges of criteria weights (Table 5) demonstrate the framework's stability. Since there is a low sensitivity to variations in criteria weights within this range, this indicates that the model produces relatively consistent outcomes despite moderate changes in the weighting scheme.

Criteria	**min**	**max**
Risky Behavior	0.22	0.4884
Knowledge	0.01	0.40
Compliance	0.01	0.38
Attitudes	0.01	0.31
Training	0.01	0.26

Table 5: *Range of weights for different criteria.*

Further specific examples of changes in criteria weights were demonstrated in Supplementary files. They show that whereas the framework has a low sensitivity to changes in the criteria weights within the ranges, its ability to discern ideal from negatively ideal solutions effectively diminishes for uniformly distributed criteria. In addition, while there is no meaningful reason to treat Risky Behavior as a beneficial factor (as such an approach would substantially deviate from the proposed framework if it were attempted), it would result in significantly different rankings.

The comparison of the TOPSIS results to the PROMETHEE-derived results (available in the Supplementary file) shows a strong agreement in identifying the lowest-performing employees, demonstrating that the framework is highly reliable in assessing poor performance, which is particularly important in the context of cybersecurity risk evaluation. A Spearman's correlation coefficient of 0.6768 between PROMETHEE and TOPSIS's rankings suggests a moderate to strong positive relationship between the results of the two methods. It indicates that the framework is fairly robust, with both methods agreeing on a substantial portion of the rankings.

3. Discussion

The results indicate that the TOPSIS-based framework for evaluating employee cybersecurity risk (TFEECR) effectively distinguishes between high-risk and low-risk employees by evaluating multiple criteria: knowledge, compliance, risky behaviors, attitudes, and training. This approach addresses the limitations of traditional single-criterion models, which often fail to capture the complexity and variability of human behavior in cybersecurity contexts.

In line with previous studies, the utility of multi-criteria decision-making (MCDM) methods in cybersecurity is reaffirmed [8, 13, 16]. For instance, Khoroshko et al. [13] highlighted the importance of incorporating multiple factors in security evaluations to provide more comprehensive and accurate assessments. Our study extends this line of research by focusing specifically on employee-related risks, emphasizing the critical role that human factors play in organizational cybersecurity.

A relevant finding is the identifiable impact of risky behaviors on an employee's cybersecurity risk profile. This aligns with the literature emphasizing the importance of mitigating human errors and non-compliant behaviors [4, 11, 14]. The strong influence of risky behaviors suggests that targeted interventions aimed at reducing such behaviors could substantially enhance organizational security.

Whereas TOPSIS itself is not a new methodology, its integration into a structured framework for the evaluation of employee-related cybersecurity risk provides originality. In addition, the study also sheds light on the practical application of TOPSIS in evaluating cybersecurity risks, demonstrating its ability to handle different criteria and provide clear, actionable rankings. This aligns with prior research that utilized TOPSIS in other security-related assessments [30, 31].

4. Conclusion

The TOPSIS-based framework for evaluating employee cybersecurity risk (TFEECR) presents a flexible, robust, and comprehensive method for assessing employee-related cybersecurity risks. It is adaptive and scalable in terms that it can be further refined to satisfy the specific needs of a research goal or organization performance with the adjustable number of examined employees. By integrating multiple criteria, such as knowledge, compliance, risky behaviors, attitudes, and training, the framework offers a holistic evaluation that addresses the complexity and variability of human factors in cybersecurity. The proposed framework bridges the gap between risk assessment and risk management, allowing an analyst to ensure a structured and transparent process of selecting risk management alternatives.

This study contributes to the growing body of research advocating for multi-criteria decision-making methods in cybersecurity evaluations. The conceptual application of TOPSIS demonstrated in this study underscores its utility in providing a more objective assessment and actionable insights, enabling organizations to identify high-risk individuals and implement targeted risk mitigation strategies.

Given the inevitable limitations of using simulated data, future research should focus on empirical validation of the TFEECR framework using real-world data to establish its effectiveness and generalizability further. Additionally, exploring the integration of other MCDM methods and enhancing the framework to include dynamic and adaptive criteria based on evolving cybersecurity threats could provide even more robust and comprehensive risk assessments.

Supplementary materials

Supplementary materials are available at:
https://doi.org/10.17605/OSF.IO/TFMXB

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New results on system of variational inequalities and fixed point theorem

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Abstract. In (Croat. Oper. Res. Rev, 13(1), 131-135, (2022).) A. Benhadid showed by a counterexample that a number of publications in the research on the variational inequalities system contains inaccurate results about applying the Lipschitz continuity concept in relation to both the first and second variables during the proof of the main theorem on the part of authors. In this paper, We suggest to applying fixed point theorem to correct the main results of some publications.

Keywords: fixed point problem, relaxed (e_1, e_2)-cocoercive mappings, variational inequalities system's with nonlinear mappings.

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1. Introduction

Consider a real Hilbert space H, with its norm and inner product represented by the symbols $\|.\|$ and $\langle ., . \rangle$, respectively. Consider a closed convex set K in H. It is commonly known that a variety of problems with transportation, fluid mechanics, boundary value, and equilibrium issues (see [2, 6, 8, 9, 11]) reduce to variational inequality situation:

$$\text{look for a point } \alpha \in K : \langle G(\alpha), v - \alpha \rangle \geq 0, \forall v \in K, \tag{1}$$

where $G : H \to H$ is a nonlinear operator on H. Variational inequality also has the benefit of allowing for the investigation of a variety of topics in the domains of physics, industry, ecology, social sciences, finance, and economics.

In summary, systems of variational inequalities offer a unified and comprehensive approach to modeling and solving complex problems with multiple interacting components or agents. They provide several advantages over individual variational inequalities, including integrated modeling, efficient solution methods, and robustness to uncertainty, making them valuable tools in various fields such as engineering, economics, and multi-agent systems.

In recent years (see [3, 4, 10, 13]), several authors have been interested in the field of system of variational inequalities and they have also used it to create new iterative algorithms for handling other pertinent problems. The system of variational inequalities that follows was established by Verma [14] in 2004. It entails determining $(\alpha, \beta) \in K^2$ such that:

$$\begin{cases} \langle \rho G(\beta, \alpha) + \alpha - \beta, v - \alpha \rangle \geq 0, \forall v \in K, \rho > 0, \\ \langle \eta G(\alpha, \beta) + \beta - \alpha, v - \beta \rangle \geq 0, \forall v \in K, \eta > 0. \end{cases} \tag{2}$$

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Huang and Noor [7] then examined and researched the topic of determining $(\alpha, \beta) \in K^2$ such that :

$$\begin{cases} \langle \rho G_1(\beta, \alpha) + \alpha - \beta, v - \alpha \rangle \geq 0, \forall v \in K, \rho > 0, \\ \langle \eta G_2(\alpha, \beta) + \beta - \alpha, v - \beta \rangle \geq 0, \forall v \in K, \eta > 0, \end{cases} \tag{3}$$

where $G, G_1, G_2 : H \longrightarrow H$ are nonlinear operators. The goal of the current work is to investigate the existence and uniqueness of solutions for the system (3) and to correct the previous results of [7] and [14], by employing the definitions corrected by Benhadid in [1]. For this reason, we think revisiting the following well-known notion.

2. Preliminaries

Definition 1. *Let G be an operator from H to H, $\lambda > 0$ is a constant with:*

$$\forall (a_1, a_2) \in H^2 : \|G(a_1) - G(a_2)\| \leq \lambda \|a_1 - a_2\| \tag{4}$$

then G is called λ-Lipschitz.

Remark 1. *If $\lambda < 1$, then G is said to be a contraction.*

Definition 2. *Let G be an operator from $H \times H$ to H, $\lambda > 0$ is a constant with:*

$$\forall v \in H, \forall (a_1, a_2) \in H^2 : \|G(a_1, v) - G(a_2, v)\| \leq \lambda \|a_1 - a_2\| \tag{5}$$

then G is called λ-Lipschitz in the first variable.

Definition 3. *Let G be an operator from H to H, $r > 0$ is a constant with:*

$$\forall (a_1, a_2) \in H^2 : \langle G(a_1) - G(a_2), a_1 - a_2 \rangle \geq r \|a - b\|^2 \tag{6}$$

then G is called r-strongly monotone.

Definition 4. *Let G be an operator from H to H, $e_1 > 0$, $e_2 > 0$ are two constants with:*

$$\forall (a_1, a_2) \in H^2 : \langle G(a_1) - G(a_2), a_1 - a_2 \rangle \geq -e_1 \|G(a_1) - G(a_2)\|^2 + e_2 \|a_1 - a_2\|^2 \tag{7}$$

then G is called relaxed (e_1, e_2)-cocoercive.

Proposition 1. *[5]. Given an element $a \in H$, $b \in K$, where $K \subset H$ is a convex closed set. then the inequality*

$$\langle b - a, x - b \rangle \geq 0, \forall x \in K \tag{8}$$

is equivalent to

$$b = P_K(a), \tag{9}$$

where P_K is a projection of H into K and satisfies:

$$\|P_K(a) - P_K(b)\| \leq \|a - b\|, \forall a, b \in H.$$

Theorem 1. *[12] Let F be a contraction on H. Then F has a unique fixed point $a \in H$, i.e $F(a) = a$.*

Finding the solution $(\alpha, \beta) \in K^2$ of (3) is identical to finding $(\alpha, \beta) \in K^2$ such that:

$$\begin{cases} \alpha = \frac{1}{2}(\alpha + P_K[\beta - \rho G_1(\beta, \alpha)]) \\ \beta = \frac{1}{2}(\beta + P_K[\alpha - \eta G_2(\alpha, \beta)]) \end{cases} \tag{10}$$

as we can readily demonstrate using Proposition 1, which is similar to the operator's fixed point problem $F : H^2 \to H^2$ define by:

$$(u,v) \to \left(\frac{1}{2}\left(u + P_K\left[v - \rho G_1(v,u)\right]\right), \frac{1}{2}\left(v + P_K\left[u - \eta G_2(u,v)\right]\right)\right) \tag{11}$$

Proof. We have, $\forall v \in K, \rho > 0, \eta > 0$

$$\begin{cases} \langle \rho G_1(\beta,\alpha) + \alpha - \beta, v - \alpha\rangle \geq 0 \\ \langle \eta G_2(\alpha,\beta) + \beta - \alpha, v - \beta\rangle \geq 0 \end{cases} \Leftrightarrow \begin{cases} \langle \alpha - (\beta - \rho G_1(\beta,\alpha)), v - \alpha\rangle \geq 0 \\ \langle \beta - (\alpha - \eta G_2(\alpha,\beta)), v - \beta\rangle \geq 0 \end{cases}$$

By using Proposition 1, we get:

$$\begin{aligned} \begin{cases} \langle \alpha - (\beta - \rho G_1(\beta,\alpha)), v - \alpha\rangle \geq 0 \\ \langle \beta - (\alpha - \eta G_2(\alpha,\beta)), v - \beta\rangle \geq 0 \end{cases} &\Leftrightarrow \begin{cases} \alpha = P_K\left[\beta - \rho G_1(\beta,\alpha)\right] \\ \beta = P_K\left[\alpha - \eta G_2(\alpha,\beta)\right] \end{cases} \\ &\Leftrightarrow \begin{cases} \alpha = \frac{1}{2}\left(\alpha + P_K\left[\beta - \rho G_1(\beta,\alpha)\right]\right) \\ \beta = \frac{1}{2}\left(\beta + P_K\left[\alpha - \eta G_2(\alpha,\beta)\right]\right) \end{cases} \end{aligned} \tag{12}$$

which means that:

$$(\alpha,\beta) = F(\alpha,\beta) \quad \square$$

3. Main result

In this section, we'll show the existence and uniqueness of solution for the problem (3).

Theorem 2. *Let $G_1 : H^2 \to H$ be relaxed (γ_1, r_1)−cocoercive in the first variable, μ_1-Lipschitzian in the first variable and λ_1-Lipschitz in the second variable. Let $G_2 : H^2 \to H$ be relaxed (γ_2, r_2)−cocoercive in the first variable, μ_2 Lipschitzian in the first variable and λ_2-Lipschitz in the second variable. If*

$$\begin{cases} r_1 - \gamma_1\mu_1^2 > \mu_1, \\ \dfrac{r_1 - \gamma_1\mu_1^2 + \sqrt{\left(r_1 - \gamma_1\mu_1^2\right)^2 - \mu_1^2}}{\mu_1^2} < \rho < \dfrac{r_1 - \gamma_1\mu_1^2 + \sqrt{\left(r_1 - \gamma_1\mu_1^2\right)^2 - \frac{3}{4}\mu_1^2}}{\mu_1^2}, \rho < \dfrac{1}{2\lambda_1} \end{cases} \tag{13}$$

$$\begin{cases} r_2 - \gamma_2\mu_2^2 > \mu_2, \\ \dfrac{r_2 - \gamma_2\mu_2^2 + \sqrt{\left(r_2 - \gamma_2\mu_2^2\right)^2 - \mu_2^2}}{\mu_2^2} < \eta < \dfrac{r_2 - \gamma_2\mu_2^2 + \sqrt{\left(r_2 - \gamma_2\mu_2^2\right)^2 - \frac{3}{4}\mu_2^2}}{\mu_2^2}, \eta < \dfrac{1}{2\lambda_2} \end{cases} \tag{14}$$

then there existe a unique solution of the system (3).

Proof. To check the result, we need to assess $\|F(x,y) - F(z,t)\|$ where $(x,y), (z,t) \in H^2$ and $\|(x,y)\|_{H^2} = \|x\|_H + \|y\|_H$.

$$F(x,y) - F(z,t) = \left(\frac{1}{2}\left(x + P_K\left[y - \rho G_1(y,x)\right]\right), \frac{1}{2}\left(y + P_K\left[x - \eta G_2(x,y)\right]\right)\right) \tag{15}$$

$$- \left(\frac{1}{2}\left(z + P_K\left[t - \rho G_1(t,z)\right]\right), \frac{1}{2}\left(t + P_K\left[z - \eta G_2(z,t)\right]\right)\right) \tag{16}$$

$$= (A,B) \tag{17}$$

First we need to evaluate:

$$\|A\| = \frac{1}{2}\left\|x + P_K\left[y - \rho G_1(y,x)\right] - \left(z + P_K\left[t - \rho G_1(t,z)\right]\right)\right\| \tag{18}$$

So,

$$\begin{aligned}
2\|A\| &= \|(x-z)+[P_K[y-\rho G_1(y,x)]-P_K[t-\rho G_1(t,z)]]\| && (19)\\
&\le \|x-z\|+\|[P_K[y-\rho G_1(y,x)]-P_K[t-\rho G_1(t,z)]]\| && (20)\\
&\le \|x-z\|+\|[y-\rho G_1(y,x)]-[t-\rho G_1(t,z)]\| && (21)\\
&\le \|x-z\|+\|y-t-\rho[G_1(y,x)-G_1(t,z)]\| && (22)\\
&\le \|y-t-\rho[G_1(y,x)-G_1(t,x)+G_1(t,x)-G_1(t,z)]\| && (23)\\
&\quad +\|x-z\| && (24)\\
&\le \|x-z\|+\|y-t-\rho[G_1(y,x)-G_1(t,x)]\| && (25)\\
&\quad +\rho\|G_1(t,x)-G_1(t,z)\| && (26)
\end{aligned}$$

From the relaxed (γ_1, r_1) −cocoercive for the first variable on G_1, we have

$$\begin{aligned}
\|y-t-\rho[G_1(y,x)-G_1(t,x)]\|^2 &= \|y-t\|^2-2\rho\langle G_1(y,x)-G_1(t,x), y-t\rangle && (27)\\
&\quad +\rho^2\|G_1(y,x)-G_1(t,x)\|^2 && (28)\\
&\le -2\rho\left[-\gamma_1\|G_1(y,x)-G_1(t,x)\|^2+r_1\|y-t\|^2\right] && (29)\\
&\quad +\|y-t\|^2+\rho^2\|G_1(y,x)-G_1(t,x)\|^2 && (30)\\
&= 2\rho\gamma_1\|G_1(y,x)-G_1(t,x)\|^2-2\rho r_1\|y-t\|^2 && (31)\\
&\quad +\|y-t\|^2+\rho^2\|G_1(y,x)-G_1(t,x)\|^2 && (32)
\end{aligned}$$

From the μ_1-Lipschitzian definition for the first variable on G_1, we have:

$$\|y-t-\rho[G_1(y,x)-G_1(t,x)]\|^2 \le \left[1+2\rho\gamma_1\mu_1^2-2\rho r_1+\rho^2\mu_1^2\right]\|y-t\|^2 \tag{33}$$

From the λ_1-Lipschitzian definition for the second variable on G_1, we have:

$$\|G_1(t,x)-G_1(t,z)\| \le \lambda_1\|x-z\| \tag{34}$$

As a result, we have:

$$\|A\| \le \frac{1}{2}\left(\|x-z\|+\theta_1\|y-t\|+\rho\lambda_1\|x-z\|\right) \tag{35}$$

where,

$$\theta_1 = \left[1+2\rho\gamma_1\mu_1^2-2\rho r_1+\rho^2\mu_1^2\right]^{\frac{1}{2}} \tag{36}$$

Similary we have:

$$\|B\| \le \frac{1}{2}\left(\|y-t\|+\theta_2\|x-z\|+\eta\lambda_2\|y-t\|\right), \tag{37}$$

where,

$$\|B\| = \frac{1}{2}\|y+P_K[x-\eta G_1(x,y)]-(t+P_K[z-\eta G_1(z,t)])\| \tag{38}$$

and

$$\theta_2 = \left[1+2\eta\gamma_2\mu_2^2-2\eta r_2+\eta^2\mu_2^2\right]^{\frac{1}{2}}. \tag{39}$$

The conditions (13) and (14) make it evident that,

$$\begin{cases}\theta_1 < \frac{1}{2},\\ \eta\lambda_2 < \frac{1}{2},\end{cases} \text{and} \begin{cases}\theta_2 < \frac{1}{2},\\ \rho\lambda_1 < \frac{1}{2}.\end{cases} \tag{40}$$

So,

$$\theta_1 + \eta\lambda_2 < 1 \quad \text{and} \quad \theta_2 + \rho\lambda_1 < 1. \tag{41}$$

Then from (35) and (37),

$$\begin{aligned}
\|A\| + \|B\| \leq & \frac{1}{2}\left(\|x-z\| + \theta_1 \|y-t\| + \rho\lambda_1 \|x-z\|\right) \\
& + \frac{1}{2}\left(\|y-t\| + \theta_2 \|x-z\| + \eta\lambda_2 \|y-t\|\right) \\
\leq & \frac{1}{2}\left[\|x-z\| + \|y-t\|\right] + \sigma\frac{1}{2}\left[\|x-z\| + \|y-t\|\right] \\
\leq & \frac{1+\sigma}{2}\left[\|x-z\| + \|y-t\|\right]
\end{aligned} \tag{42}$$

where, $\sigma = \max\left(\theta_1 + \eta\lambda_2, \theta_2 + \rho\lambda_1\right) < 1$.
So,

$$\|F(x,y) - F(z,t)\|_{H\times H} = \|A\| + \|B\| \tag{43}$$

$$\leq k \|(x,y) - (z,t)\|_{H\times H} \tag{44}$$

where $k = \dfrac{\sigma+1}{2} < 1$,
which implies that the map F defined by (11) is a contraction, using Theorem 1 then F has a unique fixed point. □

Remark 2. *Through this note, we explain and illustrate more about the two conditions (13) and (14): We set,*

$$\begin{cases} P_1(\rho) = \mu_1^2\rho^2 + 2(\gamma_1\mu_1^2 - r_1)\rho + 1 \\ P_2(\rho) = \mu_1^2\rho^2 + 2(\gamma_1\mu_1^2 - r_1)\rho + \frac{3}{4} \end{cases} \tag{45}$$

Clearly, P_1 and P_2 are two polynomials of order 2 with

$$\begin{cases} \Delta'_{P_1} = (\gamma_1\mu_1^2 - r_1)^2 - \mu_1^2 \\ \Delta'_{P_2} = (\gamma_1\mu_1^2 - r_1)^2 - \dfrac{3}{4}\mu_1^2 \end{cases} \tag{46}$$

1. *About the condition $r_1 - \gamma_1\mu_1^2 > \mu_1$.*
   We have

$$\begin{aligned}
r_1 - \gamma_1\mu_1^2 > \mu_1 > \frac{\sqrt{3}}{2}\mu_1 \Rightarrow & \begin{cases} (r_1 - \gamma_1\mu_1^2)^2 > \frac{3}{4}\mu_1^2, \\ (r_1 - \gamma_1\mu_1^2)^2 > \mu_1^2, \end{cases} \\
\Rightarrow & \begin{cases} \Delta'_{P_1} > 0, \\ \Delta'_{P_2} > 0. \end{cases}
\end{aligned} \tag{47}$$

   Which means that P_1 and P_2 have two different roots

$$\begin{aligned}
\rho_{p_1} &= \frac{r_1 - \gamma_1\mu_1^2 - \sqrt{\left(r_1 - \gamma_1\mu_1^2\right)^2 - \mu_1^2}}{\mu_1^2}, \\
\rho'_{p_1} &= \frac{r_1 - \gamma_1\mu_1^2 + \sqrt{\left(r_1 - \gamma_1\mu_1^2\right)^2 - \mu_1^2}}{\mu_1^2}, \\
\rho_{p_2} &= \frac{r_1 - \gamma_1\mu_1^2 - \sqrt{\left(r_1 - \gamma_1\mu_1^2\right)^2 - \frac{3}{4}\mu_1^2}}{\mu_1^2}, \\
\rho'_{p_2} &= \frac{r_1 - \gamma_1\mu_1^2 + \sqrt{\left(r_1 - \gamma_1\mu_1^2\right)^2 - \frac{3}{4}\mu_1^2}}{\mu_1^2}.
\end{aligned} \tag{48}$$

With:

$$0<\rho_{p_1}<\rho_{p_2}<\rho'_{p_1}<\rho'_{p_2} \tag{49}$$

2. *About the condition:* $\dfrac{r_1-\gamma_1\mu_1^2+\sqrt{\left(r_1-\gamma_1\mu_1^2\right)^2-\mu_1^2}}{\mu_1^2}<\rho<\dfrac{r_1-\gamma_1\mu_1^2+\sqrt{\left(r_1-\gamma_1\mu_1^2\right)^2-\dfrac{3}{4}\mu_1^2}}{\mu_1^2}$.

   We have :

$$\begin{aligned}
\rho'_{p_1}<\rho<\rho'_{p_2} &\Rightarrow \begin{cases} \rho_{p_2}<\rho<\rho'_{p_2} \\ 0<\rho_{p_1}<\rho'_{p_1}<\rho \end{cases} \\
&\Rightarrow \begin{cases} P_2(\rho)<0 \\ P_1(\rho)>0 \end{cases} \\
&\Rightarrow \begin{cases} \mu_1^2\rho^2+2(\gamma_1\mu_1^2-r_1)\rho+\frac{3}{4}<0 \\ \mu_1^2\rho^2+2(\gamma_1\mu_1^2-r_1)\rho+1>0 \end{cases} \\
&\Rightarrow 0<\mu_1^2\rho^2+2(\gamma_1\mu_1^2-r_1)\rho+1<\tfrac{1}{4} \\
&\Rightarrow \theta_1^2<\frac{1}{4} \\
&\Rightarrow \theta_1<\frac{1}{2}
\end{aligned} \tag{50}$$

   The same explanation with θ_2 and η.

Remark 3. *From the preceding proof, we can replace conditions (13) and (14) by a weaker condition* $\begin{cases} \mu_1^2\rho^2+2(\gamma_1\mu_1^2-r_1)\rho+1>0 \\ \theta_1+\eta\lambda_2<1 \end{cases}$ *and* $\begin{cases} \mu_2^2\eta^2+2(\gamma_2\mu_1^2-r_2)\eta+1>0 \\ \theta_2+\rho\lambda_1<1 \end{cases}$.

Remark 4. *We obtain an approximation of (α,β) via the fixed point Algorithm $X_{n+1}=F\left(X_n\right)$ i.e:*

$$\begin{cases} x_{n+1}=\dfrac{1}{2}\left(x_n+P_K\left[y_n-\rho G_1\left(y_n,x_n\right)\right]\right) \\ y_{n+1}=\dfrac{1}{2}\left(y_n+P_K\left[x_n-\eta G_2\left(x_n,y_n\right).\right]\right) \end{cases} \tag{51}$$

For any given initial points $x_0, y_0 \in H$.

4. Conclusion

In this work, we introduced and described a novel system of variational inequalities using two different operators. We suggested and presented the correct Lipschitz definition concerning the first and/or second variable in order to solve this system of variational inequalities (see [1]) via the fixed point technique. This result corrected the main result of [7], and since this new system includes the system of variational inequalities using the single operator as a special case, it also corrected the main result of [14].

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Non-cooperative inventory games for defective items and quantity discounts using strategic complementarity

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Abstract. In this paper, the mathematical analysis for obtaining the equilibrium of the inventory games under strategic complementarities, the existence of defective items, and quantity discounts have been analyzed. The inventory system consists of many buyers who order a single type of product from one supplier. They compete with each other as a player in a non-cooperative game with strategic complementary. They maximize a supermodular payoff function and take into account some fraction of defective items from a lot of the arrival products. The concept of supermodular games is used to obtain the equilibria of these problems. A new existence theorem of Nash equilibrium in a specific condition has been proved. The optimum analysis has been justified for two conditions, that is the condition without discount and another without it. The numerical computations are provided using Python programming. At the end, the numerical result shows that elements of the Nash equilibrium set can be altered when discounts are considered. A quantity discount policy can be used by the supplier to prevent players from choosing the least Nash equilibrium.

Keywords: complementarity, defective, lattice, supermodular

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1. Introduction

The ordered games characterized by strategic complementarities are extension of the ordinary game. They are often defined in a lattice structure with specific ordering relations. Then, the class of games with strategic complementarities is called supermodular games. The study of these games was initiated by Topkis [21, 22] for submodular games. The properties and the equilibrium of the supermodular games are further developed by Milgrom and Roberts [17], Vives [24], and de' Orey [5]. Moreover, Milgrom and Shannon [18] provided an explanation of monotone comparative statics in supermodular games. Topkis [23] also completed his previous result, which was implemented in them. The concepts of supermodular games also have been applied to economy, stock competitions, and inventory problems, e.g., a study on the kinds of games using supermodularity by Amir [1]; NTU supermodular games in supermodular form by Koshevoy, et al. [12]; Stock competition problem using supermdular games by Chen [4], and some application results in inventory games. Focusing on the recent results in inventory problems, supermodular game concepts have been successfully implemented in the analysis of inventory games. For instance, Cachon [2], Cachon and Netessine [3], and Lippman and McCardle [15] analyzed a multiplayer inventory using this game concept.

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In the real condition of inventory management, some products with a defect will always be found after the screening process. The defects can be caused by an imperfect production and shipment processes. Therefore, the lot size that arrives at the buyer often contains defective items with a certain defective rate. In the deterministic case, the defective rate can be determined as a fraction of defective items. Based on the result by Jaber and Bonney [8], Jaber, et al. [9], and Konstantaras et al. [11], the defective rate conforms to a logistic curve. One of the learning curves used in the application is the S-shaped logistic curve. This value depends on a parameter representing how much the shipping process is carried out by the vendor. Another important assumption in inventory management relations is a synchronization policy. Using these policies, both vendor and the buyers can optimize the cycle time. Some research results about these issues in inventory management have been proposed by some authors. Hoque [6] proposed a comprehensive analysis of the synchronization process in the supply chain with a single manufacturer and multiple buyers under the integrated assumption. Jha and Shanker [10] analyzed the same topics for defective items under some additional assumptions such as a controllable lead time and service level constraint. Mandal and Giri [16] presented an analysis of a single vendor-multi-buyer integrated inventory using a synchronization process under controllable lead time and reduction of defective items. Lin [14] also proposed the analysis of a multiplayer inventory system using a synchronization process under controllable lead time and distribution-free demand. From those last three results, it can be concluded that the synchronization assumption works well in the inventory model for defective items.

One of the famous policies to promote the product in a supply chain coordination is a quantity discount. One of the results of supply chain analysis with discount policy is presented by Li and Liu [13]. Moreover, some researchers such as Wee [25] and Huang et. al. [7] have proposed the analysis of supply chain systems for deteriorating products with discounts. The handling process for deteriorating products is similar to the handling process for a defective item in inventory management. Therefore, it can be applied in the supply chain for defective items. Despite all efforts so far, we still haven't found any records of the inventory games application in a supply chain coordination with a discount policy.

To the date, there are have been no results concerning inventory games that employ strategic complementarity to tackle a combination problem to defective items and a quantity discount policy. This study has examined this issue within the framework of a multiplayer inventory system. The system consists of a single supplier of the product and multiple buyers as a player in a one-shot game. All the buyers' concern is on the defective items contained at arrival products. The existence of the defective items is in form of a fraction of defective items and follows a learning curve. Some important assumptions in inventory games are included, such as the synchronization process between one supplier and all buyers, wholesale contracts, buyback contracts, and quantity discounts. Quantity discounts are offered by the supplier in some promotion periods. Due to the strategic complementarity, NC-supermodular games (NC-SGs) are applied to solve the inventory games. This research is performed using an analytical method and a numerical representation using the dominance principle elements applied in Phyton programming. The results described in this paper have significance for the development of the application of the supermodular game theory, especially for inventory problems with defective items. The rest of the paper is organized as follows: the methodology and data used in this research are given in Section 2. The analysis of the games including formulation, optimum analysis, and numerical computations is explained in Section 3. Finally, some conclusions and remarks are presented in Section 4.

2. Methodology and data

In this research, a systematic literature review method is used to obtain information about how far the development of supermodular game concepts and their application in real problem-solving. The development of the concept of supermodular is still quite limited. Most of them are referred to in Topkis' s works [21, 22]. However, it found that the equilibrium existence theory can still be developed by adding some conditions to ensure the existence of two Nash equilibria. Moreover, the application of supermodular games in inventory problems is also restrictive for an inventory model with limited assumptions. Therefore, there is still an opportunity for research on the application of supermodular games in inventory games with various assumptions. Based on these literature analyses, the application of supermodular games on inventory games for defective items has never been researched before. Therefore, mathematical analysis methods including analytical and numerical approaches are applied to obtain the optimum result of inventory games for defective items and quantity discounts using supermodular games. For the analytical approach, some concepts in lattice theory and supermodular games have been used. The existence theorem of Nash equilibrium using some specific conditions is proposed. Finally, the appropriate simulation data is used for the numerical test.

3. Results and discussion

3.1. Assumptions and notations

This section presents an inventory system involving multiple buyers who receive a single type of product from one supplier. The supplier is responsible for producing the products and managing the shipping process to meet the total demand of all buyers. Each buyer aims to maximize their profits through non-cooperative game strategies. It is assumed that the game is delivered under strategic complementarities conditions. The supplier is not directly involved in the games. However, the supplier controlled the game by issuing some rules and contracts for the buyers. If all buyers accept these agreements, the game can be started. Three main contracts will be offered by the supplier. The details of these contracts are presented as follows:

1. *Synchronization process*. The production cycle of the supplier would be synchronized with the ordering cycles of the buyers. Such synchronization is useful to reduce the total related cost for the entire inventory system.

2. *Wholesale contract*. The supplier charges each buyer the amount price per unit purchased.

3. *Buyback contract*. The supplier charges the buyers amount of wholesale price but pays the buyer amount of price per unit remaining at the end of the cycle on each side of the buyer. The supplier also charges the buyers a standard cost for handling the remaining product return process.

4. *Quantity discounts*. In some promotion seasons, the supplier offers a quantity discount for their products when purchased in greater numbers. However, if this policy is applied, the supplier will not apply the buyback contract and will not provide a warranty cost for the defective items found after the inspection period.

When all the buyers accept the contracts from the supplier, the games will be started immediately. In the discount season, if all the buyers agree to purchase the product with a quantity discount, they will play the games with discount properties on the payoff function. The buyers will submit the order quantity $q = \sum_{i=1}^{n} q_i$ to the supplier before the selling season. The supplier then produces and delivers these products in a single shipment process. The buyers' concern is on the existence of defective items in the form of a fraction of defective items θ_i from a lot of sizes of arrival products on each buyer's s side. Thus, these defective items would be included in the inventory games scheme. The fraction of defective items follows an S-shaped logistic learning curve ([9],[11]) for one shipment process, that is $\psi(1) = \frac{a}{g+e^{1h}}$, where $a = 70.067$, $g = 819.76$, and $h = 0.7932$. Without loss of generality, we set the value of the fraction θ_i to be the same for each buyer. After receiving the new arrival products, all buyers run the inspection process with the inspection rate s_i^x and period $\frac{q_i}{s_i^x}$. All products will be considered non-defective items until they are detected in the inspection process. Once the inspection process is fully completed, the defective items will be stored temporarily until the next shipment arrives, resulting in holding costs. There are two distinct types of holding costs: one for defective items and another for non-defective items. These defective items are not part of the buyback contract. They will be returned to the supplier who will only pay the warranty cost ζ_i per unit of defective items for each buyer. Before going into any detail, the other mathematical notations are explained in the following table.

Notations	Explananation
D	Cumulative demand.
r_i	Retail price per unit product.
p_i	Purchasing cost per unit product (as wholesale price).
A_i	Ordering cost per unit product.
F_i	Freight cost per unit remaining product.
b_i	Buyback cost per unit of remaining product at the end of the cycle.
h_i^1	Holding cost per non-defective item per unit of time.
h_i^2	Holding cost per defective item per unit of time.
I_i^1	Inventory level for non-defective items.
I_i^2	Inventory level for defective items.
T_i	Transfer payment.
$u_i^j\left(q_i\right)$	Per unit material cost (IDR) as a function of q_i; j is the number of price breaks.
Φ_i	The buyer i' s payoff function.
q	Decision variable, a positive integer. Order quantity $q = \sum_{i=1}^{n} q_i$.
P	Production rate.
w	The supplier's sales price per unit product.
h_v	The supplier's holding cost per unit product per unit time for the supplier.
I_v	The supplier's inventory level for the supplier.
A_v	The supplier's setup cost.
c_v	The supplier's standard cost for a returning process.
Φ_v	The supplier's payoff function.

Table 1: *Mathematical Notations.*

The following sections will provide an analysis of inventory games without discounts as the initial discussion. This will be followed by an examination of games that include quantity discounts. Finally, a comparison of the results from these two analyses will be presented.

3.2. Game formulation

First, it assumed that every buyer plays as a player in the non-cooperative games using strategic complementarities and no quantity discount offered by the supplier. From this section onward, the buyer(s) is called by a player(s). The supplier as the coordinator will also use the results of these games as a basis for taking optimal decisions. Suppose that the feasible strategic space $S_i \subseteq \mathbb{R}$, $i \in \{1, \ldots, n\}$ is a lattice with the usual ordering $\leq$ and *join* operation $x_i^{'} \vee x_i^{"} = \max\{x_i^{'}, x_i^{"}\}$ and *meet* operation $x_i^{'} \wedge x_i^{"} = \min\{x_i^{'}, x_i^{"}\}$, $x_i^{'}, x_i^{"} \in S_i$, such that the joint feasible strategic space for all players is formulated by $S = S_1 \times S_2 \times \cdots \times S_n$, which simply denoted by $S = \times_{i=1}^{n} S_i$. Set S is also a lattice with the component-wise order $\leq$ such that $\mathbf{x}^{'} \vee \mathbf{x}^{''} = \left(x_1^{'} \vee x_1^{"}, \ldots, x_n^{'} \vee x_n^{"}\right)$ and $\mathbf{x}^{'} \wedge \mathbf{x}^{''} = \left(x_1^{'} \wedge x_1^{"}, \ldots, x_n^{'} \wedge x_n^{"}\right)$, $\mathbf{x}^{'}, \mathbf{x}^{"} \in S$. Suppose

$\Phi_i : S \subset \mathbb{R}^n \rightarrow \mathbb{R}$ is payoff function for each buyer. When a selective joint strategy $\mathbf{x} \in S$ is played, then each player-i obtain their payoff $\Phi_i(\mathbf{x})$. For any selective joint strategy $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, $\mathbf{y} = (y_1, y_2, \ldots, y_n) \in S$, $\mathbf{x} \leq \mathbf{y}$ whenever $\Phi_i(\mathbf{x}) \leq \Phi_i(\mathbf{y})$. Furthermore, $\Phi_i : S \rightarrow \mathbb{R}$ is supermodular if for all $\mathbf{x}, \mathbf{y} \in S$,

$$\Phi_i(\mathbf{x}) + \Phi_i(\mathbf{y}) \leq \Phi_i(\mathbf{x} \wedge \mathbf{y}) + \Phi_i(\mathbf{x} \vee \mathbf{y}). \tag{1}$$

Let vector (y_i, x_{-i}) denotes the joint strategy vector with the strategy x_i of player i replaced by y_i in $\mathbf{x}$ and other components of $\mathbf{x}$ left unchanged. This notations also can be replaced by (q_i, q_{-i}) with the same meaning. All the buyers have one real valued payoff function Φ_i. These payoff is formed from two kinds of function, a reward function $g_i : S \rightarrow \mathbb{R}$, and a profit function $f_i : S \rightarrow \mathbb{R}$, such that $\Phi_i : S \rightarrow \mathbb{R}$, where $\Phi_i(q_i, q_{-i}) = g_i(q_i, q_{-i}) \cdot f_i(q_i, q_{-i})$. To elaborate on the explanation for a profit function, the inventory process in the supplier and the buyers will be explained as well as its respective costs.

The supplier carries out the production to fulfill the average demand from all buyers by following the synchronization process and under a finite production rate P $(P > D)$. Therefore, the average demand rate for each player i D_i is formulated by $D_i = \frac{q_i D}{q}$, where q_i is a replenishment quantity delivered to the player i every $\frac{q_i}{D_i}$ time units. The quantity of lot size from another player is denoted by q_{-i} which satisfies $q_{-i} = q - q_i$. The players place the same number of orders per unit of time and their order quantity lot size should be in proportion to their demand for shipment lot size. The shipment cycle time of the vendor is equal to the player's average ordering cycle time. The supplier makes one shipment process for all players simultaneously. Each buyer sells a single type of product in the selling season. After receiving the product and carrying out the inspection process, each player i will sell the non-defective product in the selling season. Each buyer knows well the information of their selling function. It is assumed that the selling function $L_i : S_i \rightarrow \mathbb{R}$ is real-valued. It is assumed that all buyers make a profit. The costs incurred as a result of the buyers' activities for the inventory management process comprise fixed ordering costs, fixed transportation costs, holding costs for non-defective items, holding costs for defective items, and transfer payment. The calculation of holding cost depends on the inventory level at the buyer's place. It takes a while until the product is sold, so the buyers need a budget for holding costs. There are two holding costs, holding costs for defective items and non-defective items. The holding cost term in the payoff function and the left inventory are calculated based on the on-hand inventory until the end of the cycle. On-hand inventory is explained in the following diagram (Figure 1)

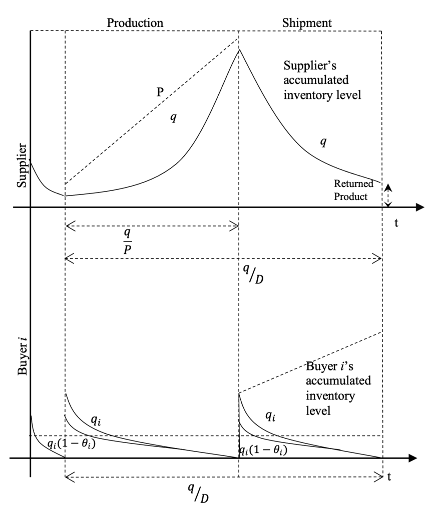

Figure 1: *Inventory Level.*

All incoming products are considered non-defective until any defects are identified at the conclusion of the inspection period. Suppose that there are $q_i\theta_i$ defective items in an arriving lot size q_i. Based on Figure 1, the average inventory level for defective items can be approximated by subtracting the accumulated defective items found during inspection time from the defective items throughout the cycle. Therefore, the average inventory level for defective items is

$$I_i^2 = \left(\frac{q_i^2(1-\theta_i)\theta_i}{D_i} - \frac{q_i^2\theta_i}{2s_i^x}\right). \tag{2}$$

The average inventory level per cycle for non-defective items is obtained from the sum of the average inventory level for non-defective items along the cycle and the defective items which are not detected yet before the end of the inspection time. Therefore, the average inventory level for non-defective items is

$$I_i^1 = \frac{q_i(1-\theta_i)}{D}\left(\frac{q_i^2\theta_i}{2s_i^x}\frac{D_i}{q_i(1-\theta_i)} + \frac{q_i(1-\theta_i)}{2}\right). \tag{3}$$

The last component of the profit function is a transfer payment for the left inventory at the end of the cycle in each buyer' s side. According to [19], the supplier should not make a profit from the remaining inventory in a news-vendor type problem. Therefore, the wholesale price per unit product must be greater than buyback cost per remaining unit, i.e.,$b_i \leq p_i$. The transfer payment process is applied to the left product of non-defective items at the end of the cycle, i.e., $I_i^1 - L_i(q_i)$. The unit cost for these transfer payment is $h_i^1 + w_i - b_i$. It is assumed that the sales revenue is less than or equal to the inventory level for non-defective items, i.e. $L_i(q_i) \leq R_i(q_i)$. To make a profit function, the revenue from selling season will be reduced by the costs. Therefore, the profit function is

$$f_i(q_i, q_{-i}) = \tfrac{D}{q(1-\theta_i)}\left(\left(p_i - h_i^1 - w_i\right)R_i(q_i) - \left(h_i^2 + w_i - \zeta_i\right)q_i^2\theta_i\left(\tfrac{1-\theta_i}{D_i} - \tfrac{1}{2s_i^x}\right)\right)$$

$$-\tfrac{D}{q(1-\theta_i)}\left(\left(A_i + F_i\right)q_i + \left(h_i^1 + w_i - b_i\right)\left(\tfrac{q_i^2(1-\theta_i)}{D}\left(\tfrac{D_i\theta_i}{2s_x^i(1-\theta_i)} + \tfrac{1-\theta_i}{2}\right) - L_i(q_i)\right)\right). \tag{4}$$

Now, we explain the second component of the payoff function, that is reward function. The reward is given by the supplier to all buyers. These reward is calculated based on the other players' left inventory for non-defective items in the end of the cycle and multiplied by some unit cost ε such that $g(q_i, q_{-i}) = \varepsilon\left(I_i^1 - L_i(q_i)\right)(q_{-i})$. Hence, the payoff function for each player is

$$\Phi_i(q_i, q_{-i}) = \varepsilon\left(I_i^1 - L_i(q_i)\right)\tfrac{D}{q(1-\theta_i)}\left(\left(p_i - h_i^1 - w_i\right)L_i(q_i) - \left(h_i^2 + w_i - \zeta_i\right)q_i^2\theta_i\left(\tfrac{1-\theta_i}{D_i} - \tfrac{1}{2s_i^x}\right)\right).$$

$$-\varepsilon\left(I_i^1 - L_i(q_i)\right)(q_{-i})\tfrac{D}{q(1-\theta_i)}\left(\left(A_i + F_i\right)q_i + \left(h_i^1 + w_i - b_i\right)\left(\tfrac{q_i^2(1-\theta_i)}{D}\left(\tfrac{D_i\theta_i}{2s_x^i(1-\theta_i)} + \tfrac{1-\theta_i}{2}\right) - L_i(q_i)(q_i)\right)\right). \tag{5}$$

All players play non-cooperative game using strategic complementarities performed in the supermodular game. These game can be notated as $G_m = (\rho, \{S_i\}_{i\in\rho}, \{\Phi_i\}_{i\in\rho})$, where $\rho = \{1, \ldots, n\}$ is number of players. A set of pure $\mathbf{x}^* \in S$ is called the Nash equilibrium of G_m if for each player i, the following condition is satisfied

$$\Phi_i\left(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^*, \ldots, x_n^*\right) \leq \Phi_i\left(x_1^*, \ldots, x_{i-1}^*, x_i^*, x_{i+1}^*, \ldots, x_n^*\right). \tag{6}$$

To guarantee the existence of the equilibrium solutions of the games, it must be assumed that S_i is a nonempty compact lattice, $\Phi_i, i = \{1, \ldots, n\}$ is the supermodular function, and also upper-semicontinuous in y_i on $S_i(x_{-i})$. After all the buyers make their optimum decision, the supplier will use these values as a reference to make the optimal decision. Using the synchronization assumption, the optimal decision is applied to run the production process, shipment, and inventory management. The supplier's profit is obtained from the purchase revenue subtracted from several costs, covering setup costs, holding costs, internal shipment costs, and transfer costs. The setup and internal shipment costs are fixed, whereas the holding cost and the transfer cost depend on the supplier's inventory level and the left product on each supplier's side, respectively. The supplier's inventory level can be approximated by subtracting all players' accumulated inventory levels from the accumulated supplier inventory level as $I_v = \frac{q^2}{2P}$. Hence, the objective function of the supplier is

$$\Phi_v(q_i) = \left(wq + \sum_{i=1}^{n}(p_i - b_i)\left(I^1 - L_i(q_i)\right)\right) - \left(A_v + F_v + h_v\frac{\left(\sum_{i=1}^{n} q_i\right)^2}{2P} + \sum_{i=1}^{n} q_i\theta_i\zeta_i\right). \tag{7}$$

The second modeling is addressed to the condition of whether all players agree to purchase the product with a quantity discount offered by the supplier. According to Wee [25], an all-units quantity discount is assumed, and the material cost can be defined as follows

$$u_i^j(q_i^j) = \begin{cases} u_i^1,\ for & m_i^1 < q_i^1 \le m_i^2 \\ u_i^2,\ for & m_i^2 < q_i^2 \le m_i^3 \\ u_i^3,\ for & m_i^n < q_i^n. \end{cases} \tag{8}$$

The discounted prices for each player follow the established relationships $u_1^i > u_2^i \cdots > u_n^i$ and $m_1, m_2, \ldots, m_n$ stands for boundaries of the incremental quantities at state 1 to n. If all players choose to purchase an all-units quantity discount offered by the supplier, then the payoff function for all players will be changed. The unit material cost will be added to Equation (5). According to the contract offered by the supplier, if this policy is applied, the supplier will not apply the buyback contract and will not provide a warranty cost for the defective items after the inspection period. The reward function component given by the supplier is preserved. Hence, the player's payoff function for the discount condition is

$$\Phi_i(q_i, q_{-i}) = \varepsilon\left(I_i^1 - L_i(q_i)\right)(q_{-i})\frac{D}{q(1-\theta_i)}\left(\left(r_i - h_i^1 - p_i\right)u^i{}_j(q_i)L_i(q_i) - \left(h_i^2 + p_i\right)q_i^2\theta_i\left(\frac{1-\theta_i}{D_i} - \frac{1}{2s_i^x}\right)\right)$$

$$-\varepsilon\left(I_i^1 - L_i(q_i)\right)(q_{-i})\frac{D}{q(1-\theta_i)}\left(\left(A_i + F_i\right)q_i + \left(h_i^1 + p_i\right)\left(\frac{q_i^2(1-\theta_i)}{D}\left(\frac{D_i\theta_i}{2s_x^i(1-\theta_i)} + \frac{1-\theta_i}{2}\right) - L_i(q_i)\right)\right). \tag{9}$$

3.3. Optimum analysis and numerical examples

In this section, the optimum analysis has been analyzed using the standard optimization method. This analysis is focuses on how the conditions that ensure the existence of the Nash equilibrium of G_m. The Nash equilibrium is determined by the supermodular game theory presented by Topkis [23]. Furthermore, the Nash equilibrium of G_m also exists if some conditions of the player's strategic space and payoff function are held. These conditions are presented in the following main theorem:

Theorem 1. *Given a non-cooperative supermodular game $G_m = (\rho, \{S_i\}_{i\in\rho}, \{\phi_i\}_{i\in\ \rho})$ such that $S_i \subset \mathbb{R}^n, i = 1, \ldots, n$ is a nonempty compact lattice. If $\phi_i(y_i, x_{-i}), i = \{1, \ldots, n\}$ is continuous in y_i for each $x_{-i} \in S_{-i}$, and $\phi_i(x)$ can be formulated as $y_i \cdot H(x_{-i})$, $S_i, i = 1, \ldots, n$ is a real bounded interval, then the equilibrium set $\mathbf{x}^* = (x_i^*, \ldots, x_n^*)$ is a nonempty complete lattice. Furthermore, if x_{-i}^b is a greatest lower bound of the set of feasible strategies of all other players such that $H\left(x_{-i}^b\right) = 0$, which $H(x_{-i})$ is a real valued function defined in real bounded interval, then there is exists the greatest Nash equilibrium $x^{*\prime}$ and the least Nash equilibrium $x^{*\prime\prime}$.*

Proof. By following the proof by Topkis [23], it can be verified that G_m has two equilibria, the largest Nash equilibrium and the smallest Nash equilibrium. Because of $S_i \subset \mathbb{R}^n$, $i = 1, \ldots, n$ is a nonempty compact lattice, then $S = S_1 \times \cdots \times S_n$ is a nonempty compact lattice. For each $i = 1, \ldots, n, \phi_i\left(., x_{-i}\right)$ is both upper semicontinuous and lower semicontinuous at the same time. There exist $\overline{y}_i \in S_i\left(x_{-i}\right)$ such that $\phi_i\left(., x_{-i}\right)$ hits a maximum at $\overline{y}_i$ such that $\overline{y}_i = argmax_{y_i \in S_i(x_{-i})} \phi(y_i, x_{-i})$. Moreover, there is $\underline{y}_i$ such that $\phi_i\left(., x_{-i}\right)$ reaches a minimum point at $\underline{y}_i$ such that $\underline{y}_i = argmin_{y_i \in S_i(x_{-i})} \phi(y_i, x_{-i})$. Since G_m is supermodular game it holds that $\phi_i(y_i, x_{-i}), i = 1, \ldots, n$ is supermodular in y_i on S_i for each $x_{-i} \in S_{-i}$ and $\phi(y_i, x_{-i})$ have an increasing differences in (y_i, x_{-i}) for each $i = 1, \ldots n$. Furthermore, for each $x_{-i} \leq x'_{-i}$ and y'_i,

$$\phi_i\left(y_i, x'_{-i}\right) - \phi_i\left(y_i,\ x_{-i}\right) \leq \phi_i\left(y'_i, x'_{-i}\right) - \phi_i\left(y'_i, x_{-i}\right). \tag{10}$$

Based on Inequality (10), if other players can obtain a higher payoff according to their choice of strategy, then player i can obtain a higher payoff accordingly. If all opposing players ($n-1$ player) of player i do not choose a strategy $x_{-i} = x_i^b$, or there exist at most l opposing players ($l < n-2$) which play strategy $x_{-i} = x_{-i}^b$, then player i will choose $y_i = x_i^a, x_i \leq x_i^a, x_i \in S_i$ as an optimum response to obtain the higher payoff. Hence, $\left(x_i^a, x_{-i}^a\right)$ is the greatest Nash equilibrium. □

The form $\phi_i\left(y_i, x_{-i}\right) = y_i \cdot H\left(x_{-i}\right)$ is an important condition to ensure that two Nash equilibrium exist or may the least equilibrium not exist. The term $H\left(x_{-i}\right)$ must be presented in a single form of strategy without involving any constant terms. Since $S_i, i = 1, \ldots, n$ is a real bounded interval, i.e. $S_i \subset \mathbb{R}$, then the upper bound and the lower bound of S_i is the greatest and the least Nash equilibrium of G_m, respectively.

All known direct numerical methods to obtain the Nash equilibrium focus on the games with two-player only. Although some of them have been tried to apply in a game with three players, it is not easy to apply to every case and algorithm. One of the famous methods in a non-cooperative game is the dominance principle of the elements of the payoff matrix. It is designed for a noncooperative game with two players. Although has been developed for three players, it is still hard to compute. Accordingly, the numerical example is presented based on the games with two players (buyers) only. Because all players apply the strategy with complementarity properties, then the component-wise ordering, which is not a complete ordering, is still appropriate with the steps of the algorithms of the principle dominance. Although there exist pairs of strategies that can not be compared using component-wise ordering, e.g., (2,3) and (2,4), they can be ignored using the complementarity properties in a supermodular game. In these cases, if a pair of strategies $(q_1, q_2) = (2, 3)$ is being chosen, so a pair of strategies (2,4) will not be selected.

Next, some numerical illustrations of G_m for two players' cases are presented. Numerically, the Nash equilibrium for a class of non-cooperative games can be obtained using the dominance property principle of the payoff matrix. As explained in the previous section, the payoff method is commonly available for two-player cases only. This was also explained in detail by [20]. Furthermore, an algorithm can be developed in Python based on the pseudocode construction presented by [20]. Overall, the pseudocode uses a brute-force approach to find the Nash equilibrium points. Each combination of player strategies is examined to ensure that no player can improve their payoff by changing strategies. This method works well for games with a limited number of strategies; however, it becomes inefficient when players have numerous strategies due to the considerable computational time required. In our research, we developed an algorithm in Python based on the pseudocode by [20], as we found it suitable for the case we are discussing. The following is the algorithm in Python.

```python
import numpy as np

# Define the bounds of the strategic space for Player I and Player II
player_I_strategies = np.array([1, 2, 3])  # Example strategies for Player I
player_II_strategies = np.array([1, 2, 3])  # Example strategies for Player II

# Payoff matrices for each player
payoff_player_I = np.zeros((len(player_I_strategies), len(player_II_strategies)))
payoff_player_II = np.zeros((len(player_I_strategies), len(player_II_strategies)))

# Input payoffs for each combination of strategies
for i, strategy_I in enumerate(player_I_strategies):
    for j, strategy_II in enumerate(player_II_strategies):
        # Manual input or using a function
        payoff_player_I[i, j] = int(input(f"Enter payoff for Player I at strategy ({strategy_I}, {strategy_II}): "))
        payoff_player_II[i, j] = int(input(f"Enter payoff for Player II at strategy ({strategy_I}, {strategy_II}): "))

# Determine Nash Equilibrium
nash_equilibrium = []

for i in range(len(player_I_strategies)):
    for j in range(len(player_II_strategies)):
        # Check if Player I gets the maximum payoff for the current strategy of Player II
        player_I_best_response = np.max(payoff_player_I[:, j]) == payoff_player_I[i, j]
        # Check if Player II gets the maximum payoff for the current strategy of Player I
        player_II_best_response = np.max(payoff_player_II[i, :]) == payoff_player_II[i, j]

        # If neither player can improve their payoff
        if player_I_best_response and player_II_best_response:
            nash_equilibrium.append((player_I_strategies[i], player_II_strategies[j]))

# Print the Nash Equilibrium results
if nash_equilibrium:
    print("Nash Equilibrium found at strategies:")
    for eq in nash_equilibrium:
        print(f"Player I: {eq[0]}, Player II: {eq[1]}")
else:
    print("No Nash Equilibrium found.")
```

Listing 1: *Python code to find Nash Equilibrium.*

Suppose that each player agrees to the following joint strategy space.

$$S = \{\mathbf{x} | q_1 = q_2, q_2 \in [1,3)\} \cup ([3,40] \times [3,40]) . \tag{11}$$

The sales function for the first and second buyers is defined by $L_i : S_i \to \mathbb{R}, i \in \{1,2\}$, where $L_1(q_1) = K\left(\frac{3}{16}q_1^2 + \frac{1}{8}q_1\right)$, and $L_2(q_2) = K\left(\frac{3}{16}q_2^2 + \frac{1}{10}q_2\right)$, $K = \frac{1}{D}\left(\frac{D_i\theta_i}{s_i^x} + (1-\theta_i)^2\right)$. For numerical test, we consider the following the data: $D_1 = 45, D_2 = 35, s_1^x = 100$ unit product per unit time, $s_2^x = 95$ unit product per unit time, $p_1 = 400, p_2 = 450, A_1 = A_2 = 0.003, F_1 = F_2 = 0.001, h_1^1 = h_2^1 = 1.5, h_1^2 = h_2^2 = 2, w_1 = w_2 = 45, \zeta_1 = \zeta_2 = 1$, $b_1 = b_2 = 15$, and $\varepsilon = 0.5$. The fraction of the defective items for all buyers is the same value and expressed using an S-shaped logistic learning curve for one shipment process, i.e.

$\theta_i = \psi(1) = \frac{70.067}{819.76+e^{1.0.7932}} = 0.085, i = 1,2$. First, if one of the player play a strategy $q_i \in [1,3)$, then the other player must choose the same strategy. Therefore, a set $[1,3)$ generate the Nash equilibrium of which $(1,1)$ is the least Nash equilibrium. Suppose all buyers play a strategy contained in interval $[3,40]$, then each player can be free to choose their strategy. The dominance principle of the payoff matrix is applied to determine the Nash equilibrium of G_m. For the numerical test, it takes 38 possible strategies for each player, e.g.,$q_i = 3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,$ $31,32,33,34,35,36,37,38,39$, and 40. Therefore, the payoff matrix with 1444 elements is obtained. Using these data, we obtain the Nash equilibrium $(q_1^*, q_2^*) = (40,40)$. Hence, the highest and the least Nash equilibrium is $(q_1^*, q_2^*) = (1,1)$ and $(q_1^*, q_2^*) = (40,40)$, respectively. If (q_1, q_2) selected by all players, the first and second buyers earn $\Phi_1(40,40) = 3713.033$ in 1000 IDR and $\Phi_2(40,40) = 4057.583$ in 1000 IDR as the payoff, respectively. The existence of the defective items still preserved the ordering relation in the strategic space. The supplier investigates the optimal decision by applying the synchronization principle. It means that the supplier takes $(q_1^*, q_2^*) = (40,40)$ as the reference to determine his optimal decision, $q^* = 40+40 = 80$. Given the following data for the supplier (all values are in 1000 IDR): $A_v = 20, F_v = 1, h_v = 3$, and $P = 100$, then the largest and smallest profits for the supplier is $\Phi_v(1,1) = 69.1$ in 1000 IDR, and $\Phi_v(40,40) = 3872.29$ in 1000 IDR. Next, the numerical test is considered in the case when an all-units quantity discount is assumed. First, the unit material cost for each player is given by

$$u_1^j(q_1^j) = \begin{cases} u_1^1 = 400 \quad in \quad 1000IDR,\ for \quad 0 < q_1^1 \le 15 \\ u_1^2 = 300 \quad in \quad 1000IDR,\ for \quad 15 < q_1^2 \le 30 \\ u_1^3 = 200 \quad in \quad 1000IDR,\ for \quad 30 < q_1^3. \end{cases} \tag{12}$$

and

$$u_2^j(q_1^j) = \begin{cases} u_2^1 = 450 \quad in \quad 1000IDR,\ for \quad 0 < q_2^1 \le 15 \\ u_2^2 = 350 \quad in \quad 1000IDR,\ for \quad 15 < q_2^2 \le 30 \\ u_2^3 = 250 \quad in \quad 1000IDR,\ for \quad 30 < q_2^3. \end{cases} \tag{13}$$

The other parameters are used to illustrate the analytical results: $D_1 = 45, D_2 = 35, s_1^x = 100$ unit product per unit time, $s_2^x = 95$ unit product per unit time, $r_1 = 400, r_2 = 450, A_1 = A_2 = 0.003, F_1 = F_2 = 0.001, h_1^1 = h_2^1 = 1.5, h_1^2 = h_2^2 = 2, p_1 = p_2 = 45, \zeta_1 = \zeta_2 = 1$, and $\varepsilon = 0.5$. When a quantity discount is applied, both Player 1 and Player 2 will not choose $(q_1, q_2) = (1,1)$ as a strategy profile. In otherwords, a strategy $(q_1, q_2) = (1,1)$ is not Nash equilibrium. Based on the form of the payoff function in Equation (9), the higher the unit material cost, the greater payoff obtained by each player. Due to the use of strategic complementarity and the form of the unit material cost offered by the supplier for each player, then all players will purchase the product which is the third unit material cost, i.e.u_1^3 for Player 1 and u_2^3 for Player 2. Based on joint strategy space in Equation (11), $(q_1^*, q_2^*) = (40,40)$ is selected as the Nash equilibrium. Hence, only a single Nash equilibrium would be obtained when a quantity discount is offered by the supplier. Each player will not purchase the product in a small number. Furthermore, it can be used as a new justification that the composition of Nash equilibrium of supermodular games can be altered using appropriate policy. In that case, a quantity discount policy can be applied by the supplier to prevent the players from choosing the least Nash equilibrium.

4. Conclusion

In this research, a new application of supermodular games for inventory games with strategic complementarities due to the existence of defective items has been proposed. All players are concerned about the existence of defective items which follow the logistic learning curve. According to analytical and numerical results, the concept of supermodular games can be used to obtain the optimum value of inventory games when the players use the strategic complementary, concerned about the existence of the defective item. To guarantee the presence of both the highest and lowest equilibrium, the reward function, which is based on the remaining inventory of the other players, can be utilized as a component of the buyers' payoff function. This result is also appropriate with our main existence theorem of the equilibrium. According to the numerical results, a quantity discount policy can be used by the supplier to prevent players from choosing the least Nash equilibrium. For further research, the same research can be conducted for inventory games with more complex assumptions and other policies.

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Calculation of ecological land-footprint – based on the input-output model and focusing on the imported commodities

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Abstract. The ecological footprint has been a crucial ecological indicator for more than two decades, and the methodology for calculating it has developed significantly over the years. However, some issues and shortcomings still need to be addressed and specified further. This paper focuses on the embedded land requirements of imported commodities in input-output modelling approaches. We propose a refined model to overcome the shortcomings of two former models. Our model quantifies the embedded ecological land-footprint of imported commodities and their allocation between direct final consumption and production. In addition, we allocate the latter again among final consumption and exports using the framework of linear algebra and matrix arithmetic. We also propose ways of extending the model to overcome the general but misleading assumption in the literature that imported commodities have an equal per unit ecological footprint to domestic products, an approach that is based on the idea that trading partners have the same technological background.

Keywords: ecological land-footprint, imported commodities, input-output model

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1. Introduction

Our methodology used needs to be explained: input-output modelling is used to show the ecological land-footprint defined by the land use approach. Given that one of the by-products of the Leontief input-output model is the land area used, it is suitable for defining a static ecological land-footprint. Since its first publication in the early 1990s, the ecological footprint has become the most popular ecological indicator [23, 28]. Its purpose is to determine the amount of land that nations use (expressed in global hectares) to satisfy the long-term needs of their populations while measuring the human pressure on ecosystems. In this context, it can be considered as an absolute measure of sustainability [11, 25]. Over time, the methodology has developed, and there are approaches that deal with the ecological footprint at regional, individual, and even organisational levels [22], which enrich the sustainability debate with new perspectives. The calculation of a static ecological footprint with an input-output model consists of the following steps: 1. Select the economic sectors and activities of the country to be examined by researchers who want to include them in their calculations. 2. Collect production data for each sector, including inputs (such as energy, water, and raw materials) and outputs (such as products and waste). 3. Organise the data into an input-output table that describes the dependencies between the different sectors and the number of inputs and outputs. 4. Supplement the input-output table with various data needed to calculate the

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footprint, such as the quantity of carbon dioxide emissions or energy use. 5. Calculate the footprint using the data in the table and the data added in the previous step, which measures the ecological impacts caused by economic sectors. The paper contains the following sections: Section 2 presents the literature review and the examined models. In section 3, we present a refined method for calculating ecological footprint with the input-output model, and section 4 concludes with the results of the paper.

2. Literature review

The concept of the ecological footprint has become one of the most widely used indicators of land use, alongside other popular terms like the circular economy and sustainable development. Although many models are less commonly applied today, they could be revived with modern enhancements. One such model is the original Leontief model, which did not initially include environmental factors but is suitable for calculating land use. Leontief later incorporated environmental aspects by presenting a static input-output model with pollution factors [16]. In this study, we calculate the land-related ecological footprint, building on two influential studies in the field. Our approach constructs the footprint based on vectors of final consumption, exports, and directly imported products consumed within a given period. The studies by Bicknell et al. [3] and Ferng [10] advanced the methodology of input-output models, providing the foundation for further developments. Csutora and Vetőné Mózner [5] applied a similar static input-output model to determine carbon emissions at the industry level, while Dobos [8] offered methodological critiques of these models. Leontief's model [17] provides a comprehensive analysis of the circular economy and continues to gain relevance today [14, 18, 19, 29]. While Leontief originally introduced the static input-output model with environmental factors, recent works have attempted to make the static footprint model dynamic [6]. Input-output models can be used to approach the ecological footprint in both static and dynamic forms, and in this study, we focus on the static calculation method. Lenzen et al. [15] examined the environmental relevance of input-output models, especially in relation to ecological footprint calculations, while Wiedmann and Barrett [30] reviewed the ecological footprint indicator, discussing its advantages, limitations, and the role of input-output models. We focus particularly on land use models, such as those explored by Hubacek and Sun [13], Ferng [10], Bicknell et al. [3], and Eder and Narodoslawsky [9]. Several studies, including Hubacek and Giljum [12] and Turner et al. [27], applied static methods to calculate the ecological footprint, focusing on pollution and land use. They concluded that importing products from industrialised countries significantly contributes to developing countries' natural environment degradation. Wiedmann and Barrett [30] also explored various interpretations and calculation methods for the ecological footprint, emphasising how input-output models can improve footprint calculations. Furthermore, research by Gao and Tian [21] examined greenhouse gas emissions in China from 1980 to 2030, using ecological footprint and input-output models to identify key factors driving the growth of emissions. Bringezu et al. [4] studied sustainable production and biofuels, using input-output models to assess biofuels' ecological and social impacts compared to other energy sources. These examples demonstrate the broad application of ecological footprint calculations and input-output analysis in the literature. Numerous studies provide specific calculations of ecological footprint. For instance, Abbood et al. [1] used an input-output model to calculate the carbon and energy footprint of the U.S. manufacturing system, quantifying the life-cycle impacts of U.S. manufacturing activities in the context of international trade. Specific case studies include analyses of ecological footprint for Malaysia [2], Finland [20], Japan [26], and China [31]. The application of ecological footprint in various regions illustrates the adaptability of input-output models to different environmental and economic contexts. Additionally, the Higgins Sustainable Growth Index incorporates the ecological footprint as a key variable, providing another practical application of the concept [24]. In the next section, we analyse two influential studies on ecological land-footprint: Bicknell et al. [3] and Ferng [10]. Bicknell's study was the first to model land use by considering whether it is driven by imports or domestic production. The model calculates the share of land allocated to consumption, including three types of land associated with imports: land for final direct consumption, land entering production for consumers, and land indirectly transferred from imports to exports. Ferng's work builds on this by introducing a land multiplier to improve the calculations [10]. The key difference between the two studies is that Ferng models land as a consumable resource rather than a primary input, and they differ in how they allocate imported land between final consumption and exports. One limitation of these models is the inability to partition sectoral imports between final consumption and exports linearly.

3. A refined method for calculating ecological footprint with the input-output model

When calculating the ecological footprint using empirical data, a relevant problem is the lack of data for the other countries. Therefore, it is proposed to substitute the data of the country under study for the data of the other countries, thereby facilitating the calculations. Given that the ecological footprint aims to capture the total direct and indirect resource use embodied in final consumption, input-output models provide an ideal accounting framework. Bicknell's model [3] was the first to address how to allocate imported land between final consumption and exports. Ferng [10] refined this by introducing a land multiplier, but even this method does not fully resolve the issues. In our study, we describe the mathematical contributions of these works and propose improvements to better account for the embedded footprint of imports, addressing differences in technological backgrounds and their impact on ecological footprint calculations. The basis of input-output analysis is a set of sectoral disaggregated economic accounts that identify the inputs and outputs of each sector. This method allows for the quantification of interdependencies within the economy, using straightforward mathematical routines to trace all direct, indirect, and induced resource use within consumption [16, 21]. The methods of Bicknell [3] and Ferng [10] differ in their approach to calculating the land requirements of imports used indirectly for consumption and exports. Bicknell allocates imports across sectors based on final consumption and exports, while Ferng reduces total output by exports, leaving the relevance of imports for production ambiguous. The first case presented by Bicknell [3] treats land as a resource, where specific land use is calculated per unit of output. Ferng's model assumes intra-sectoral exchange and incorporates land use in addition to exports. For a standard inter-sectoral model, this distinction is crucial, particularly in whether land use is considered in terms of the import or export columns. A more detailed comparison of the two models can be found in the work by Dobos and Tóth-Bozó [7]. In the next section, we will present the mathematical background for calculating land footprint based on input-output models. This background builds on the economic assumptions of previous studies, but offers a more precise mathematical formulation.

Our proposal is constructed in a framework shown in Table 1. The table differs from Table 3 of paper [7] as the imports are shared between the economies (countries) with that our hypothetical economy is in commercial relations - such type of modelling can be found in regional economics. Let us assume that there n sectors in the national economy are modelled and that there are m different further economies that have a commercial relationship with our economy. As before, the gross output of the economy is denoted by x, c vector is the final consumption, exp vector represents the exports, while X is cross-sectoral use. The vector l shows the land demand of the economy. The X_{imp_i} matrix shows the commodities imported from the ith economy and used by the different sectors of our economy, while the c_{imp_i} vector represents commodities imported from the ith economy to the final consumption of our national economy.

Vectors v and v_c show added values. The imp_i vectors represent commodities imported in total to the examined national economy, i.e., $imp_i = X_{imp_i}1 + c_{imp_i}$, where vector 1 denotes the summing vector, each element of that is one.

	Sectors	Final consumption	Export	Total output, import	Land (ha)
Sectors	X	c	exp	x	l
Country 1	X_{imp_1}	c_{imp_1}		imp_1	
Country 2	X_{imp_2}	c_{imp_2}		imp_2	
...	...	...		...	
Country m	X_{imp_m}	c_{imp_m}		imp_m	
Added value	v	v_c			
Total output	x				

Table 1: *An input-output table for a national economy with foreign trade.*

Where

- X_{imp_i} is the output from imported goods from the ith country, $i = 1, 2, ..., m$;
- c_{imp_i} is the final consumption from imported goods from the ith country, $i = 1, 2, ..., m$;
- v is the total added value vector with n dimension;
- v_c is the added value for final consumption with n dimension;
- x is the total output in the economy;
- imp_i is the total import from the ith country, $i = 1, 2, ..., m$;
- l is the total land use in hectare;

Then construct the model input coefficients for the economy and imports:

$$A = X\langle x\rangle^{-1}, \quad A_{imp_i} = X_{imp_i}\langle x\rangle^{-1}, \qquad (i = 1, 2, ..., m) \tag{1}$$

With these coefficient matrices, we can obtain the following equations describing the economy:

$$x = Ax + c + exp, \quad imp_i = A_{imp_i}x + c_{imp_i}, \qquad (i = 1, 2, \ldots, m) \tag{2}$$

In this context, the model is essentially the same as the one of the Table 1, as $X_{imp} = \sum_{i=1}^{m} X_{imp_i}$,$c_{imp} = \sum_{i=1}^{m} c_{imp_i}$, we did not do anything other than arranging imports according to their sources.

Considering the ecological footprint, as we can see, the land multiplier in the Ferngian sense is only known to our modelled economy (for the other economies, we should be aware of the input-output models of each national economy). If we would know the other land multipliers as well, they would follow the following general form: $\langle l_i\rangle(I - A_i)^{-1}$, where matrix A_i is the sectoral relationship balance of i nation's economy and vector l_i is the land demand of that economy.

Based on that, the imports used by the economy we use can be summarised thus:

$$\sum_{i=1}^{m}\langle l_i\rangle(I - A_i)^{-1}imp_i = \sum_{i=1}^{m}\langle l_i\rangle(I - A_i)^{-1}A_{imp_i}x + \sum_{i=1}^{m}\langle l_i\rangle(I - A_i)^{-1}c_{imp_i} \tag{3}$$

This expression is the same as in the earlier models, if $A = A_i$ and $l = l_i$, meaning that every economy is homogeneous in the sense that they have the same technology matrix and same area. In this case, the following relationship is fulfilled: $imp = \sum_{i=1}^{m} A_{imp_i} x + \sum_{i=1}^{m} c_{imp_i}$, which means that we sum up the imports regardless of their origin. Further analysis is conducted on this aggregate model where $A_{imp} = \sum_{i=1}^{m} A_{imp_i}, c_{imp} = \sum_{i=1}^{m} c_{imp_i}$.

So the model is $x = Ax + c + exp$ and $imp = A_{imp}x + c_{imp}$.

And one may solve this model to obtain the total output: $x = (I - A)^{-1}c + (I - A)^{-1}exp$ and $imp = A_{imp}(I - A)^{-1}c + A_{imp}(I - A)^{-1}exp + c_{imp}$. Here, the first system of equations describes the domestic production, while the second one shows imports as a function of final domestic consumption, exports and products directly imported for final consumption. We can determine the land demand by multiplying the domestic equations by the diagonal matrix of the domestic area, while applying the land multiplier to the imported products.

With this calculation:

$$\langle l \rangle x = \langle l \rangle (I - A)^{-1}c + \langle l \rangle (I - A)^{-1}exp \tag{4}$$

the following expressions are obtained:

$$\langle l \rangle (I-A)^{-1}imp = \langle l \rangle (I-A)^{-1}A_{imp}(I-A)^{-1}c + \langle l \rangle (I-A)^{-1}A_{imp}(I-A)^{-1}exp + (I-A)^{-1}c_{imp} \tag{5}$$

Thus, we have five layers of land requirements:

- $\langle l \rangle (I - A)^{-1}c$: domestic land requirements of final consumption,
- $\langle l \rangle (I - A)^{-1}exp$: domestic land requirements of export,
- $\langle l \rangle (I-A)^{-1}c_{imp}$: embedded land requirements of direct imports serving final consumption,
- $\langle l \rangle (I - A)^{-1}A_{imp}(I - A)^{-1}c$: embedded land requirements of indirect imports (for the production sector serving final consumption),
- $\langle l \rangle (I - A)^{-1}A_{imp}(I - A)^{-1}exp$: embedded land requirements of indirect imports (for the production of exported products).

In the light of these categories, we can classify land requirements into two groups: (1) domestic demand for land use that is linked to final consumption, and (2) land demand for foreign consumption that is attached to exports and thus does not appear in domestic consumption. This latter land demand can be counted as a transit item, since after being imported, it will leave the country as export and will not enter the country and its economy, so it will not be part of the land requirements of the country.

We will now compare our **proposed** approach with the two previous models, as shown in the 2nd table.

	[3]	[10]	Proposed model
Domestic land requirements of final consumption	$l(I-A)^{-1}c$	$\langle l\rangle(I-A)^{-1}c$	$\langle l\rangle(I-A)^{-1}c$
Embedded land requirements of direct imports serving final consumption	$l(I-A)^{-1}\langle c_{imp}\rangle$	$\langle l\rangle(I-A)^{-1}c_{imp}$	$\langle l\rangle(I-A)^{-1}c_{imp}$
Embedded land requirements of production	$l(I-A)^{-1}A_{imp}\langle x\rangle$	–	–
Embedded land requirements of indirect import (for the production serving final consumption	$l(I-A)^{-1}A_{imp}\cdot$ $\cdot\langle x\rangle\langle c\rangle\langle c+exp\rangle^{-1}$	$\langle l\rangle(I-A)^{-1}\cdot$ $\cdot A_{imp}(x-exp)$	$\langle l\rangle(I-A)^{-1}\cdot$ $\cdot A_{imp}(I-A)^{-1}c$
Domestic land requirements of exports	–	$\langle l\rangle(I-A)^{-1}exp$	$\langle l\rangle(I-A)^{-1}exp$
Embedded land requirements of indirect imports (for the production of exported products)	–	–	$\langle l\rangle(I-A)^{-1}A_{imp}\cdot$ $\cdot(I-A)^{-1}exp$

Table 2: *The categories to be captured by the papers.*

It is immediately apparent that paper [10] did not specify the land requirements related to the import demand of exports, which is a barrier for precisely determining the land requirements of the imported products used for production. As

$$(x-exp)-c=Ax\geq 0 \tag{6}$$

the weighted land demand in our proposed model will be lower than the figure arrived at by the Ferngian model. A numerical comparison of the three models in Table 2 can be found in work [7].

Having expanded the refined Leontief's input-output model by taking into account the land footprint, we summarise the obtained results in the conclusion section. These results enable a more accurate footprint calculation.

4. Conclusion

A slight inaccuracy found in previous modelling has now been corrected. By our method we have calculated lower footprints, which significantly modifies the earlier overestimation. The use of input-output model for calculating static ecological footprint may pose several mathematical difficulties. These include handling larger input-output tables, solving nonlinear equations, and analysing and visualising multidimensional data. In addition, the accuracy of the model may be questionable, as environmental impacts are often difficult to measure precisely and require estimation. Another difficulty of the input-output model is that it does not consider temporal changes, meaning it cannot model dynamic effects. Finally, the model may be sensitive to the quality and accuracy of input data, so the accuracy of the results depends on the quality of input data. Our result is that we have created a newly formulated land footprint calculation model that features an input-output model. By our method we calculated lower footprints in work [7], which significantly modifies the earlier overestimation. We have also shown that imports could be generally managed to calculate the footprint, but this would require a massive calculation since all the technical coefficients of each national economy concerned would need to be known. The other authors assume that different commodities are produced under the same technological conditions, and thus the same land requirements, in every country. In a subsequent analysis, the presented model is appropriate for calculating the ecological footprint of a real economy (furthermore, it can also be used for calculating other footprints, like the carbon footprint). Another possible extension of the model is afforded by the dynamism of the footprints, that is, the impact of accumulation and investment processes on the footprint. For this, the dynamic version of the Leontief model can serve as an appropriate method.

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Optimizing maritime routes: A multi-method analysis from Shanghai to Vladivostok

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Abstract. This research analyzes the marine route plan from Shanghai to Vladivostok utilizing Dijkstra's algorithm, Markov chain analysis, game theory, and congestion analysis. Dijkstra determines the route through Busan and Hungnam as the shortest, with a total distance of 2114 kilometers and minimal travel time. The Markov chain analysis supported the designated path by demonstrating greater transition probabilities compared to other routes, so establishing it as the most probable option. Experts in game theory, particularly on the Nash equilibrium, demonstrated that cooperation significantly reduced operating expenses. Further congestion research corroborated that the Shanghai-Busan-Hungnam-Vladivostok route offers a cost advantage, and even with the inclusion of congestion, the route remains less expensive. The study collectively advocates for the consideration of distance, likelihood, collaboration, and congestion while selecting the optimal maritime route, hence enhancing efficiency in maritime logistics.

Keywords: congestion analysis, dijkstra's algorithm, game theory, maritime route optimization, markov chain analysis.

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1. Introduction

The symmetry of directions in maritime transportation is seen significant in today's worldwide context, particularly due to the expansion of many shipping options. The continuous expansion of globalization and market liberalization need effective management of these channels to conserve time, reduce costs, and enhance dependability. Numerous variables provide significant challenges to the shipping sector, including fuel pricing, port size discrepancies, environmental impacts, and political instability, among others. Consequently, several characteristics delineate international freight, necessitating enhancements to optimize its advantages for all parties concerned.

Analyzing route options within an intermodal transport network using mathematical models and game theory is the focus of this study. In order to accomplish a marine route mapping without disturbances, we use Dijkstra's algorithm on the issue graph, do Markov chain analysis on the probabilities, and incorporate aspects of the Nash equilibrium and the prisoner's dilemma. The key channel linking the main East Asian economic players is the maritime route

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between Shanghai (China), Busan (South Korea), Hungnam (North Korea), and Vladivostok (Russia), and this route is the focus of the analysis.

In congestion games, players take turns using shared resources, like roadways, which get more congested as more players join the fray. Specifically, we examine how port congestion in cities like Vladivostok and Shanghai affects shipping route costs using congestion game theory. Each shipping business acts as a participant, influencing his or her cost, due to the congestion at these important ports. This method provides theoretical groundwork for evaluating and controlling the effects of congestion on shipping costs.

For graph-based representations of geographical models, such as transportation networks, Dijkstra is employed for path search in order to discover the best possible route. On the other hand, factors impacting shipping time and expense are involved, therefore there are inherent uncertainties in marine logistics. Markov chain is a method for explaining and evaluating uncertainty that depicts state transfers from one port to another along with the associated probability. Thus, the optimal pathways may be analyzed twice using deterministic and stochastic methods.

The incorporation of game theory into the model enhances and conjugates these domains, as game theory addresses strategic decisions in a competitive setting. Shipping corporations, port authorities, and regulators are all important players, yet they may have conflicting interests. The strategic interactions taking place among marine logistics players can be better understood by analyzing the prisoner's dilemma and the Nash equilibrium. In contrast to Nash Equilibrium, which describes a situation in which no one has any reason to seek change that would be better for them than everyone else involved, the prisoner's dilemma explores the possibilities of coordinated and collective cooperation in divided and conflicting scenarios. This synthesis of approaches tackles several important concerns:

1. How can we adjust or reroute marine routes to account for unpredictable time and money?
2. What impact does this optimization method have on strategic relationships among stakeholders?
3. How might game theory enhance cohesion in marine logistics decision-making?

This research contributes to the literature on fleet route planning by integrating Dijkstra's algorithm, the Markov chain model, and congestion game theory. The study aims to provide more specific recommendations for policymakers and various stakeholders, particularly owners and managers of shipping companies, as well as participants in global commerce engaged in the effective control and optimization of maritime supply chain management.

2. Literature review

Logistics and transportation networks have optimization components, rendering their study a complicated domain that incorporates several mathematical disciplines. The integration of Dijkstra's algorithm, Markov chains, and game theory has significantly enhanced the resolution of route selection issues, particularly in congestion games.

2.1. Dijkstra's algorithm in network optimization

Dijkstra's method, introduced by Dijkstra [5], is a foundational algorithm in graph theory that efficiently determines the shortest path between two nodes in a network. This algorithm has been widely employed in several transportation and logistics scenarios, as well as in maritime and intermodal transportation networks. Route optimization has become essential, particularly in selecting appropriate routes, since this aids in minimizing expenses and transit duration. The algorithm's route selection process is particularly crucial in dynamic scenarios, such as fluctuating congestion and time delays, which are prone to frequent changes.

2.2. Markov chains in dynamic routing

Markov chains provide a stochastic method for evaluating systems that progress across time, where the future state of the system relies solely on the current state, independent of its historical context [6, 7]. Route optimization has employed Markov chains to simulate the stochastic performance of transportation systems, enabling dynamic routing based on current conditions. For example, [10] used Markov models to enhance the traffic management of real-time marine navigation, demonstrating that these models can accommodate the stochastic and dynamic attributes of supply chain networks.

2.3. Game theory and congestion games

Game theory applications, such as Nash equilibrium and non-cooperative games, have proved significant in addressing problems related to route selection and network congestion. In [12], the analysis of strategic interactions inside competitively organized systems in non-cooperative games was conducted, establishing a foundational framework. Congestion games are a subset of non-cooperative games, particularly pertinent in transportation networks when several users use the same connection, resulting in interference and a conflict of interest [11, 9]. [1, 13] presents studies on the evolution of cooperation, while [2] authored a 400-page tome on game theory that elucidates the behaviors of various actors inside these networks. [8] further examines the critique of game theory by emphasizing its relevance to the analysis of network congestion and the rational selection of routes. Recent literature has focused on simulating congestion games in transportation networks using game theory. For example, [3] investigated the use of game theory to enhance route selection in logistics management and analyzed the optimization of maritime routes under uncertainty using game theory. These investigations demonstrate that game theory, particularly congestion games, serves as an effective framework for modeling congestion and offers several mechanisms for its management in transportation systems.

2.4. Integrating techniques

Employing Dijkstra's algorithm in conjunction with Markov chains and game theory guarantees the best selection of routes inside transportation networks. These approaches will facilitate the examination of both deterministic and stochastic attributes of network optimization, taking into account constant and changing variables as well as the interactions between leaders and followers. These strategies synergistically enhance one another, yielding more effective and precise solutions to routing challenges, particularly in congested networks where standard optimization techniques struggle to achieve optimal results[4]. The review of literature on Dijkstra's algorithm, Markov chains, and game theory, particularly in the context of congestion games, illustrates the evolution and use of these techniques in addressing modern transportation challenges. As additional participants enter the market and supply chains get increasingly complex, the capacity to maintain and enhance these strategies will be crucial for optimizing routes.

3. Methodology and data analysis

3.1. Sea ports: distances and traveling time

The study concentrates on enhancing marine connections between Shanghai (China) and Vladivostok (Russia) through essential seaports: Busan in the Republic of Korea, Osaka in Japan, and Hungnam in the Republic of Korea. Commence by gathering comprehensive data on the distance of each port along the maritime corridor from Shanghai (China) to Vladivostok (Russia) at a vessel speed of 10 knots, utilizing credible sources such as marine databases, shipping

logs, and geographic information systems (GIS). This data encompasses the subsequent information. Precise distances between each port pair (e.g., Shanghai to Busan, Busan to Osaka, and Osaka to Vladivostok), assuming a vessel speed of 10 knots. Historical and real-time statistics regarding transit durations and frequency of marine routes connecting these ports. Environmental and operational aspects affecting marine transport, including meteorological conditions and seasonal fluctuations. The nodes are assigned as follows:
Node 1 is Shanghai (China), node 2 is Busan (South Korea), node 3 is Osaka (Japan), node 4 is Hungnam (North Korea), and node 5 is Vladivostok (Russia). The distances and travel durations are displayed in Table 1 and may also be seen in Figure 1.

Paths	Distances (km)	Time(minutes)
$1 \to 2$	911	3000
$1 \to 3$	1469	4740
$2 \to 3$	689	2220
$2 \to 4$	583	1920
$3 \to 4$	1187	3840
$3 \to 5$	1511	4920
$4 \to 5$	620	2040

Table 1: *Distances in km and time in minutes are calculated.*

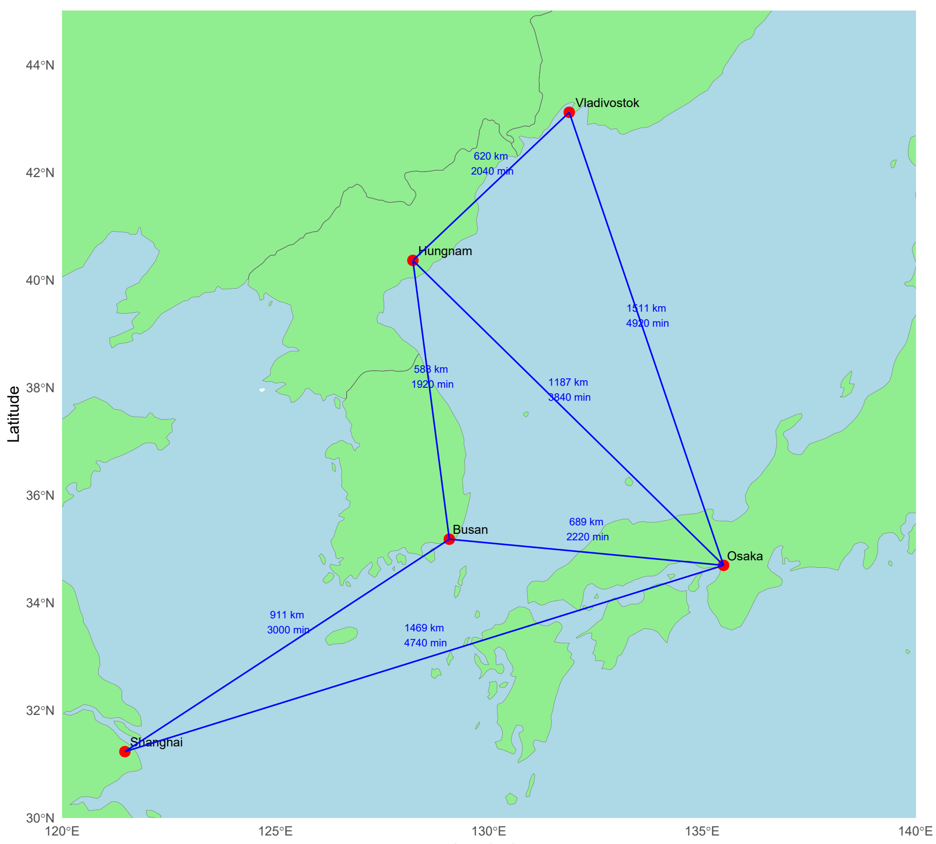

Figure 1: *Map of selected sea ports with their routes.*

4. Dijkstra's algorithm

1. A graph $G = (V, E)$, where V is the set of vertices and E is the set of edges.

2. The direct distance between any two nodes is given and all the distances are non-negative ≥ 0 .

3. In case there is no way of getting from i to j to directly, we set $d_{ij} = \infty$

4. The algorithm proceeds by assigning to all nodes with a label either temporary or permanent.

   Step 0: Node 1 is first designated as permanent and its distance is established at 0, denoted as 0*. Initialize the distances to all other nodes as infinity (∞). Establish a permanent node and include the source node with a priority of zero.

   Step 1: Keep going until either every node has been visited or the shortest path to the target node has been found.

4.1. Markov chain analysis

1. **States**: Establish the states using our methodology as a foundation. In international transportation, for example, states might stand in for various ports or nodes located in different nations (e.g., Shanghai (China) port to Vladivostok (Russia) port).

2. **Transition probabilities:** These denote the likelihood of state transitions. These might be calculated with assumptions or historical data. One might standardize time and distance to articulate probability if inclusion is desired.

   Distance-based probabilities: A potential approach is to convert distances into probabilities (e.g., inversely proportional to distance) when states are ports or nodes, with transitions dictated by the distance separating them. The total equals 1 for each initial node.
   $p_{ij} = \frac{1}{d_{ij}} / \sum_k \frac{1}{d_{ik}}$, where d_{ij} is the distance from node i to node j.

   Time-based probabilities: If changes depend on trip times, turn them into possibilities (for example, in a way that is opposite to time). That adds up to 1 for each node that starts.
   $p_{ij} = \frac{1}{t_{ij}} / \sum_k \frac{1}{t_{ik}}$, where t_{ij} is the travel time from node i to node j.

3. **Normalization:** Make sure that the sum of the chances of moving from one state to another is one. This shows the Markov property, which says that the future state depends only on the present state and not on the events that happened before.

4. **Transition probability matrix:** A matrix shows the chance of each change between any two states in the system. Put these chances into a transition matrix. Each entry p_{ij} in the matrix represents the chance of going from state i to state j. It should be in the i^{th} row and the j^{th} column.

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1j} & \cdots & P_{1k} \\ P_{21} & P_{22} & \cdots & P_{2j} & \cdots & P_{2k} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{i1} & P_{i2} & \cdots & P_{ij} & \cdots & P_{ik} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nj} & \cdots & P_{nk} \end{bmatrix} \tag{1}$$

where the cumulative transition probability from state i to all other states must equal 1.

4.2. Game theory application

4.2.1. Modeling stakeholder interactions

To determine the optimal routes for vessels traveling from Shanghai to Vladivostok, we employ game theory. This elucidates how two competing factions determine their course of action. The groupings under consideration are:
Company A: Ensures that deliveries are consistently made on time while slashing operational expenses.
Company B: Increasing Shipping's market share while maintaining peak efficiency is the company's top priority.

4.2.2. Prisoner's dilemma setup

In the prisoner's dilemma, two stakeholders must negotiate their interactions within a specific context. In decision-making, both sides must select one of two potential tactics.
Optimize route (strategy O): Selecting a strategy that prioritizes route optimization facilitates cost and time savings.
Maintain status quo (strategy M): Entails adhering to established pathways and methodologies.

4.2.3. Constructing the payoff matrix

In the realm of stakeholder decision-making, the reward matrix functions as a navigational tool. It illustrates the consequences of various options, particularly for the distance and duration of maritime travel. This is the methodology employed to construct that map:

1. **Define routes and costs:** Establish the routes to pursue and ascertain their associated costs.

   Route 1: The route begins in Shanghai, proceeds to Busan, traverses to Osaka, and ultimately arrives in Vladivostok. The voyage consists of many segments: first from Shanghai to Busan, including 911 kilometers; secondly from Busan to Osaka, adding 1469 kilometers; and lastly from Osaka to Vladivostok, comprising 689 kilometers, resulting in a cumulative distance of 3069 kilometers. The voyage from Shanghai to Busan requires 3000 minutes, followed by an additional 4740 minutes from Busan to Osaka, and lastly 3840 minutes from Osaka to Vladivostok, culminating in a total duration of 11580 minutes.

   Route 2: The route starts in Shanghai, proceeds to Busan, continues to Hungnam, and concludes in Vladivostok. The voyage starts in Shanghai and proceeds to Busan, encompassing 911 kilometers, followed by a segment from Busan to Hungnam, which adds 583 km, and concludes with the leg from Hungnam to Vladivostok, measuring 620 km, resulting in a cumulative distance of 2114 km. The travel from Shanghai to Busan requires 3000 minutes, followed by an additional 1920 minutes from Busan to Hungnam, and lastly, 2040 minutes from Hungnam to Vladivostok, culminating in a total of 6960 minutes.

2. **Determine payoffs:** (Identify benefits) The advantages of route optimization may be succinctly expressed as follows:

   Operational cost savings: Achieving savings by reducing distances and time.

   Competitive advantage: Enhancing market share and efficiency through optimized routes.

Stakeholder B / Stakeholder A	Optimize route (O)	Maintain current approach (M)
Optimize route (O)	(Cost1, Advantage1)	(Cost2, Advantage2)
Maintain current approach (M)	(Cost3, Advantage3)	(Cost4, Advantage4)

Table 2: *The payoff matrix.*

where

Cost i; $i = 1, 2, 3, 4$ represent costs linked to route choices.
Advantage j; $j = 1, 2, 3, 4$ signify changes, in advantages or market share.

4.2.4. Analyzing equilibrium

A Nash equilibrium is determined to uncover stable strategies in which neither party can enhance their payout by unilaterally altering their policies. This study pertains to the identification of equilibrium points: What are the most probable tactics for each stakeholder? The impact of these tactics on the optimization of weights and dimensions in curricula, together with their competitive positioning.

4.2.5. Strategic implications

1. **If both companies optimize routes:** Compare cost savings of two companies with/without joint optimization of routes.
2. **If both maintain status quo:** Evaluated impact of need to continue depending on established routes.
3. **Mixed strategies:** In cases when one firm optimizes and another stays the same, what changes or responses may be possible.

4.3. Congestion game theory

A congestion model scenario with several participants, including various shipping businesses, that share restricted resources like port facilities. This study focuses on the impact of congestion on the cost structures of each participant and how these costs influence route selection. In the context of transportation routes between ports, each route functions as a participant. The participants may choose their routes, with certain ports perhaps experiencing congestion while others do not. The primary resources for pooling, as perceived by stakeholders, are the port capabilities of Shanghai, Busan, Osaka, Hungnam, and Vladivostok. The utilization of a shared port is marked by congestion, resulting in increased costs. Each participant or shipping business choose their best route from Shanghai to Vladivostok, considering the congestion conditions at the ports. Strategies encompass choosing routes that reduce congestion at ports or alternatively accepting the expenses associated with overloaded ports. The payoff for each participant is contingent upon the cumulative marine cost, the port service costs, and the congestion cost associated with the selected route. The cost function for player i traversing route r is defined as:

$$C_i(r) = c_{\text{maritime}} \times d_r + \text{Port Fees}_r + \text{Congestion Cost}_r \tag{2}$$

where

c_{maritime} = Cost per nautical mile
d_r = Distance of the route (in nautical miles)
Port Fees$_r$ = Handling and other port charges
Congestion Cost$_r$= Additional cost due to congestion at ports along the route.

The congestion cost of the ports is determined by assessing the average number of users that engage with the respective port. As other players select the same port, the congestion fee increases. This is executed in a way analogous to the preceding example, whereby a function delineates the impacts of congestion, which elevate delays and operational costs inside the system. For instance, if k players utilize port j, the congestion cost Congestion Cost$_j$ may be represented as:

$$\text{Congestion Cost}_j = \alpha \cdot k_j \tag{3}$$

where
α = Congestion factor (a constant representing the increase in cost per additional player)
k_j = Number of players using port j

5. Result and discussion

5.1. Dijkstra's algorithm

Identify the most efficient and quickest maritime route between the ports of Shanghai and Vladivostok, considering distances and transit durations. The most direct route, spanning 2114 km, was determined using Dijkstra's algorithm to go from Shanghai to Vladivostok via Busan and Hungnam. The algorithm's deterministic method efficiently calculated the minimal distances between nodes, providing a key tool for route selection.

1. Shortest path from China to Russia: [Shanghai (China) → Busan (South Korea) → (Hungnam) North Korea → Vladivostok (Russia)] The distances traveled from other communities are 911 km, 583 km and 620 km, collectively 2114 km, with vessel speed, 10 knots. The time traveled from other communities are 3000 minutes, 1920 minutes, 2040 minutes, and collectively 6960 minutes, with vessel speed, 10 knots.

2. Alternative path from China to Russia: [Shanghai (China) → Osaka (Japan) → Vladivostok (Russia)] The distances traveled from other communities are 1469 km, and 1511 km, collectively 2980 km, with vessel speed, 10 knots. The time traveled from other communities are 4740 minutes and 4920, and collectively 9660 minutes, with vessel speed, 10 knots.

5.2. Markov chain analysis

We employed Markov chain analysis to examine transition probabilities between nodes (sea ports) and to determine the most probable path from Shanghai to Vladivostok, incorporating stochastic elements into our route selection process. This is based on the unequivocal findings obtained via Dijkstra's method. We determined the probability of transitioning from one location to another utilising historical data and shipping information. This provided us with a serendipitous perspective on identifying the optimal path. The principal conclusions from the Markov chain research, which presents the distance-based probabilities transition matrix P, are as follows:

$$P = \begin{bmatrix} 0 & 0.617 & 0.383 & 0 & 0 \\ 0 & 0 & 0.470 & 0.530 & 0 \\ 0 & 0 & 0 & 0.890 & 0.110 \\ 0 & 0 & 0 & 0 & 1.00 \end{bmatrix} \tag{4}$$

This matrix displays the likelihood of transitioning from one state to another in a Markov chain. All probabilities in the row total to 1 (or to the input probabilities if some internal states lack complete transition sums), indicating that the probabilities are appropriately normalised. Ultimately, from a practical perspective, it facilitates the computation of transition probabilities in decision-making, route selection, and forecasting the system's future state. The most likely route may be characterised as the path where each transition between states has the highest likelihood of happening, with a vessel speed of 10 knots.
Shanghai (China) → Busan (South Korea) → (Hungnam) North Korea → Vladivostok (Russia)
with probabilities: $p_{12} = 0.617$, $p_{24} = 0.530$, $p_{45} = 1.000$, the total product probabilities = 0.3270, and time-based probabilities transition matrix P:

$$P = \begin{bmatrix} 0 & 0.612 & 0.388 & 0 & 0 \\ 0 & 0 & 0.463 & 0.537 & 0 \\ 0 & 0 & 0 & 0.562 & 0.438 \\ 0 & 0 & 0 & 0 & 1.00 \end{bmatrix} \tag{5}$$

The most likely path is characterised by transitions between states that exhibit the highest chance of occurrence. The most probable path is
Shanghai (China) → Busan (South Korea) → Hungnam (North Korea) → Vladivostok(Russia)
with probabilities: $p_{12} = 0.612$, $p_{24} = 0.537$,$P_{45} = 1.000$, and the total Product probabilities = 0.32

5.3. Game theory analysis

This research aims to establish an academic basis for assessing route optimisation options through the use of prisoner's dilemma game theory, with the strategic actions of two stakeholders (shipping company A and shipping company B) as rationale. The stakeholder's strategies are: (i) Optimize route (O) (implement advanced optimization techniques), and (ii) maintain status quo (M) (continue with existing routing practices).

	Shipping Co. B: Optimize (O)	Shipping Co. B: Maintain (M)
Shipping Co. A: Optimize (O)	(2000, 2000)	(2500, 2200)
Shipping Co. A: Maintain (M)	(2200, 2500)	(2700, 2700)

Table 3: *The payoff matrix.*

Payoffs are expressed in USD, while operating expenses are assessed for each strategic combination. The essence of Nash equilibrium is in the mutual selection of optimization strategies by both parties involved. This yields the minimal operational expenses, with each side receiving 2000 USD. Neither stakeholder has a rationale to choose an alternative approach, since the ideal joint optimization plan provides the most advantageous cost structure, improves market positioning, and facilitates a reduction in total operating expenses.

5.4. Congestion game analysis

This section analyses the congestion game for the two alternative marine routes from Shanghai (China) to Vladivostok (Russia). The research examines the idea of overall costs, encompassing marine transport costs, port handling expenses, and congestion costs.
Route 1: Shanghai (China) → Osaka (Japan) → Vladivostok (Russia).

Route 1	Distance (km)	Distance (nautical miles)	Maritime cost (€)	Port handling (€)
Shanghai to Osaka	1,469 km	794 miles	1,588	400
Osaka to Vladivostok	1,511 km	816 miles	1,632	550

Cost component of route 1	Amount (€)
Total maritime cost	3,220
Total Port handling cost	950
Total cost without congestion	**4,170**
Congestion cost (10%)	417
Total cost with congestion	**4,587**

Table 4: *Distance, time, and cost calculations for Shanghai-Osaka-Vladivostok route.*

Route 2: Shanghai (China) → Busan (South Korea) → Hungnam (North Korea) → Vladivostok (Russia).

Route 2	Distance (nautical miles)	Maritime cost (€)	Port handling (€)
Shanghai to Busan	491 miles	982	450
Busan to Hungnam	315 miles	630	350
Hungnam to Vladivostok	335 miles	670	550

Cost component of route 2	Amount (€)
Total base maritime cost	2,282
Total Port handling cost	1,350
Total cost without congestion	**3,632**
Congestion Cost (10%)	363.20
Total cost with congestion	**3,995.20**

Table 5:*Distance, time, and cost calculations for Shanghai-Busan-Hungnam-Vladivostok route.*

A congestion game aims to assess the overall expenses associated with two distinct marine routes from Shanghai (China) to Vladivostok (Russia). The total expense of marine transport is thus determined by maritime transport costs, port management costs, and congestion costs.
Route 1: Shanghai (China) → Osaka (Japan) → Vladivostok (Russia).

The distances between the stops on the Shanghai-Osaka-Vladivostok route are 1,469 km, while those on the Osaka-Vladivostok-Shanghai route are 1,511 km.The total marine cost amounts to €3220, with the port handling cost being €950. The aggregate expense excluding congestion is €4,170. With the inclusion of a 10% congestion cost of €417, the total cost with congestion increases to €4,587.
Route 2: Shanghai (China) → Busan (South Korea) → Hungnam (North Korea) → Vladivostok (Russia).

The distances for the Shanghai-Osaka-Vladivostok route are 1,469 km, while the Osaka-Vladivostok-Shanghai route measures 1,511 km.The total marine cost amounts to €3220, while the port handling fee is €950. The aggregate expense excluding congestion amounts to €4,170. With the inclusion of a 10% congestion fee of €417, the total cost with congestion increases to €4,587.

This research indicates that Route 2, which has more port handling stops, is less expensive than Route 1, including both direct expenses and congestion considerations. The effectiveness study findings encompass the relative efficiency of marine routes between Shanghai and Vladivostok, utilizing Dijkstra's algorithm, Markov chain analysis, game theory, and congestion game analysis. These findings align with Dijkstra's shortest path between Busan and Hungnam, as well as the Markov chain exhibiting a high transition probability for this particular route. From a game theory viewpoint, it may be noted that cooperation in optimizing work routes can lead to a significant reduction in operating expenses; the Nash equilibrium demonstrates that all parties benefit. Analysis of congestion games reveals that, despite the greater number of ports serviced along the Shanghai-Busan-Hungnam-Vladivostok route compared to the Shanghai-Osaka-Vladivostok route, its congestion costs are relatively lower. This focus highlights the need of utilizing distance, likelihood, cooperative approach, and congestion variables as criteria for route selection.

6. Conclusion

The examination of marine routes from Shanghai to Vladivostok, using Dijkstra's algorithm, Markov chain analysis, game theory, and congestion game analysis, indicates a preference for the Shanghai-Busan-Hungnam-Vladivostok route. This route provides the shortest distance, higher transition probabilities, and reduced overall cost when accounting for congestion. Strategies from cooperative game theory are employed to enhance efforts in maintaining cost efficiency and presenting evidence that demonstrates effective optimization as a means of reducing costs. Consequently, the designated route of Shanghai-Busan-Hungnam-Vladivostok is crucial for players seeking to achieve operational and economic improvements in marine logistics.

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A profit maximization model for sustainable inventory under preservation technology and linear time-dependent holding cost

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Abstract. This paper presents sustainable inventory model with constant deterioration rate when holding cost is time dependent. Demand rate is price and stock dependent. Preservation technology and carbon emission are considered for more sustainable approach. Shortages are allowed and are fully backlogged. In addition, a mathematical model is constructed to maximize the total profit function and the concavity is shown using three-dimensional graph. A numerical experimentation is carried to compute the total profit and the order quantity. To validate the proposed model, the sensitivity analysis is conducted for the total profit function as well as the order quantity. Results and observations are also discussed along with managerial insights.

Keywords: carbon emission, inventory, preservation technology, shortages, sustainable;

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1. Introduction

One of the greatest gifts we can give to next generation is a life worth living. A sustainable approach means using the world resources by thinking of future. An inventory model is not possible without carbon emission but it can be sustainable using preservation technology to prevent deterioration and wastage. [9] introduced an inventory model with time varying demand and holding cost whereas [17] formed a perishable inventory model with stochastic demand supply but the effect of carbon emission and deteriorations was missing. In this paper, preservation technology investment is made to counter the effects of deterioration on inventory with time and stock dependent demand. Carbon emission cost is also considered to make this model more environment friendly. In present world scenario, with everything becoming uncertain sustainability is the answer for the times ahead.

Different types of technologies are being discussed to make more energy efficient systems. A study by [10] showed that by investing in energy efficient technologies earlier, may result in more economic benefit with the flow of time. Many types of articles associated with preservation technologies investment (PRT) have been published recently. [25] discussed an Economic Order Quantity (EOQ) model with PRT and partial advance payments whereas [19] proposed a production model under PRT with salvage value.

The upcoming part of the paper is systematically divided into various Sections. Section 2 of this paper contains the literature review of the study whereas symbols and assumptions are showed in Section 3. Model development is constructed in Section 4 with the solution algorithm. A numerical experimentation is carried out in Section 5 with 3D graph. Sensitivity analysis, results and managerial insights are shown respectively in Sections 6 and 7 of this paper. The proposed study is concluded in Section 8.

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2. Literature review

[8] introduced an inventory model with constant demand rate. They found the minimal cost for inventory model with deteriorating items with variable rate of deterioration and known demand. [7] showed that inventory level is also an important parameter in demand rate of inventory management whereas [1] stated that pricing also plays an impactful part in determining demand rate. Deterioration is imminent and it always results in loss of profit which is why it became an important part of inventory management. [9] formulated a deteriorating inventory model with time dependent demand rate and holding cost to minimize the total cost. [21] considered two parameter Weibull distribution deterioration rate with stock and price dependent demand rate in EOQ model.

[15] developed an inventory model with constant holding cost and PRT. Completely and partially backlogged shortages was considered with two cases of demand function dependent on price and stock. [23] proposed an inventory model with price-time dependent demand and non-linear holding cost. [12] formulated an inventory model with parabolic holding cost and salvage value was also considered to reduce the wastage and total cost. [29] extended the work of [15] with carbon tax policy and carbon cap-trade policy. [5] optimized the delivery pattern to balance the workload of transport and warehouse.

[13] developed an inventory model for imperfect product with carbon emissions and investment in green technology. [18] discussed an inventory model with price and stock dependent demand rate to maximize the return on inventory management expense. To reduce the effect of global warming, several types of carbon emission norms are introduced by regulatory bodies. [16] optimized an inventory model for price dependent demand rate with carbon emission. [6] examined the learning effect on EOQ model with deteriorated items under carbon emission effect. [2] generated a multi-item inventory model with partially backlogged shortages to explore the effects of reliability.

[4] studied an inventory model with time and price dependent demand rate, non-instantaneous deterioration rate following the three-parameter Weibull distribution to find the optimal selling price. [24] proposed a model with price and stock dependent demand rate to increase the profit with more greening efforts. [26] generated an inventory model with time dependent quality demand under PRT by keeping in mind of dairy products. [28] introduced price based PRT in an inventory model to reduce carbon emissions. [3] discussed an interval valued inventory model with price dependent demand rate and buy now pay later scheme for payments.

This paper correlates time dependent holding cost with deteriorating items and PRT is made to counter deterioration and for a sustainable approach. The summary of recent work is represented in Table 1 below.

Literature	**Demand Rate**	**Holding Cost**	**Shortages**	**PRT**
[27]	Time dependent	Fixed	Partially backlogged	✓
[9]	Time dependent	Linear	Partially backlogged	×
[14]	Time dependent	Linear	Partially backlogged	×
[15]	Price-stock dependent	Fixed	Completely \ Partially backlogged	✓
[12]	Time dependent	Parabolic	Without shortages	×
[11]	Stock dependent	Fixed	Partially backlogged	×
[20]	Time dependent	Fixed	Without shortages	×
[22]	Time dependent	Fixed	Completely backlogged	×
[3]	Price dependent	Fixed	Without shortages	×
This paper	Price-stock dependent	Linear	Completely backlogged	✓

Table 1: *Summary of recent research.*

3. Symbols and assumptions

The employed symbols in the model are summarized below in Table 2.

Symbols	Meaning of the symbols	Symbols	Meaning of the symbols
n	Ordering frequency	A	Ordering cost (\$/cycle)
t_1	Time when shortage occurs (unit time)	D_τ	Deteriorated products (units)
T	Cycle length (unit time)	S	Maximum inventory level (units)
$\lambda(\alpha)$	Deterioration after PRT (units/time unit)	Q	Order Quantity (units)
λ_0	Deterioration rate (units/time unit)	SR	Sales revenue (\$/time unit)
δ	Sensitivity parameter for PRT	PC	Purchase cost (\$/time unit)
β	Stock dependent parameter for the demand function	HC	Holding cost (\$/time unit)
c	Cost price (\$/unit)	SC	Storage cost (\$/time unit)
p	Selling price (\$/unit)	DC	Deterioration cost (\$/time unit)
d	Deteriorating cost (\$/unit)	OC	Ordering cost (\$/time unit)
α	PRT cost (\$/unit/ time unit)	PTC	PRT cost (\$/time unit)
c_h	Holding cost (\$/unit/time unit)	CEC	Carbon emission cost (\$/time unit)

Table 2: *List of Symbols.*

The following assumptions are made in the inventory model.

(i) Lead time is zero.

(ii) Demand rate ($D(p,t)$) depends up on price and stock [15, 29], as follows:

$$D(p,t) = \begin{cases} D(p) + \beta I(t), & 0 \le t \le t_1 \\ D(p), & t_1 \le t \le \frac{T}{n} \end{cases}, \tag{1}$$

where $D(p)$ is demand function dependent on price and $I(t)$ is inventory level at time t. Here, $D(p) = \frac{a}{p^b}$, where a and b are non-zero positive real numbers, called the demand scale, and the price sensitive parameter respectively.

(iii) Holding cost C_h is linearly time dependent [9]

$$c_h = a_0 + a_1 t \tag{2}$$

where, a_0, $a_1 > 0$ are holding scale parameters.

(iv) The deterioration rate after preservation technology [15] is

$$\lambda(\alpha) = \lambda_0 e^{-\delta\alpha} \tag{3}$$

where, λ_0 is deterioration rate in the absence of preservation technology investment and δ is preservation technology parameter and λ_0, $\delta > 0$. The effect of preservation technology on deterioration rate is shown graphically in Figure 1.

(v) Carbon emission is considered and calculated by dividing it into two factors, fixed and variable. A tariff on carbon emission is applied to calculate the carbon emission cost.

(vi) Shortages are completely backlogged.

(vii) Single item is considered.

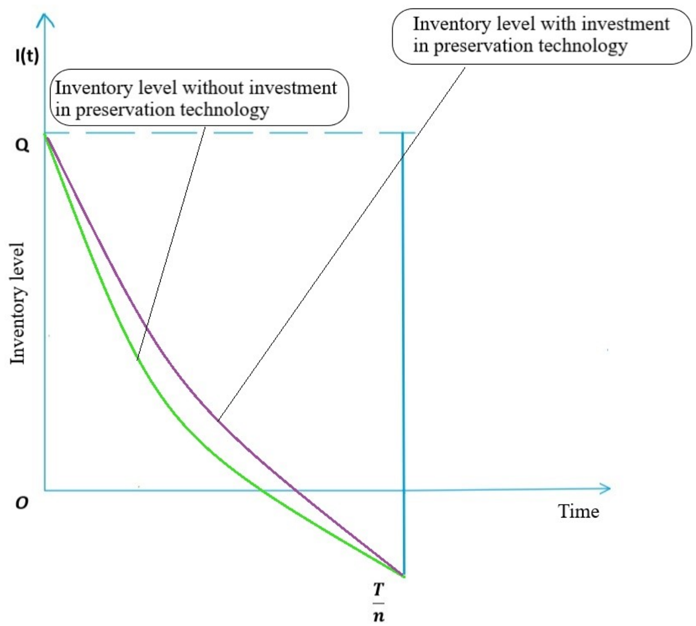


Figure 1: *Effect on inventory level with preservation technology.*

The behaviour of the inventory model is shown graphically in Figure 2.

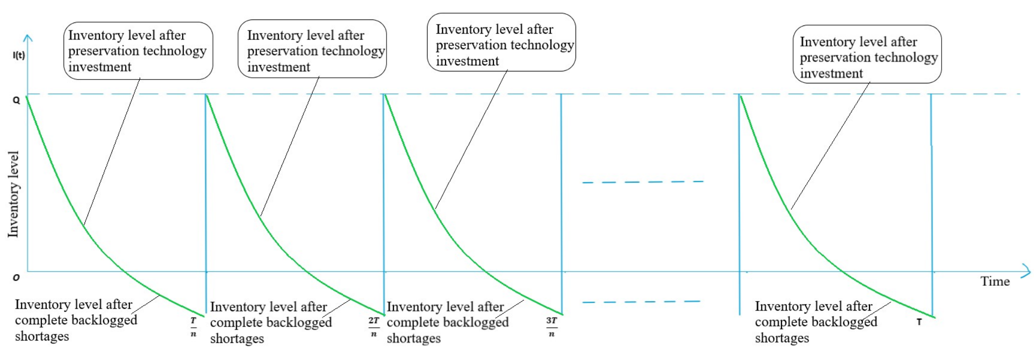


Figure 2: *Representation of inventory model with fully backlogged shortages.*

4. Model development and solution procedure

This model optimizes the value of total profit function by finding the optimal values of t_1 and T. For the inventory level (Figures 1-2), there are two differential equations: Equation (4) and Equation (5). Here, the Equation (4) holds when the inventory level is non-zero. On the other side, Equation (5) holds when the inventory level is replenished. Let $t = t_1$ when the inventory level replenishes. The concerned differential equations are presented by Equations (4) and (5) as follows

$$\frac{d(I(t))}{dt} + \lambda(\alpha)I(t)) = -D(p,t), \quad 0 \leq t \leq t_1 \tag{4}$$

$$\frac{d(I(t))}{dt} = -D(p), \quad t_1 \leq t \leq \frac{T}{n} \tag{5}$$

with the boundary conditions $I(0) = S$ when the inventory level is maximum, and $I(t_1) = 0$ when the shortage happens.

The solution of the above equations is given by

$$D(p,t) = \begin{cases} \frac{D(p)}{\lambda(\alpha)+\beta}(e^{(\lambda(\alpha)+\beta)(t_1-t)} - 1), & 0 \leq t \leq t_1 \\ D(p)(t_1 - t), & t_1 \leq t \leq \frac{T}{n} \end{cases}, \tag{6}$$

Applying the boundary conditions to Equation (9), the inventory reaches maximum level (S) which is determined as

$$S = I(0) = \frac{D(p)}{\lambda(\alpha) + \beta}(e^{(\lambda(\alpha)+\beta)t_1} - 1) \tag{7}$$

Products affected by deterioration (D_τ) in the interval $[0, t_1]$ is evaluated as

$$D_\tau = S - \int_0^{t_1} D(p,t)dt \tag{8}$$

The solution of Equation (8) is given by

$$D_\tau = \frac{D(p)}{\lambda(\alpha) + \beta} \left(e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha) + \beta)t_1 + \frac{\beta}{\lambda(\alpha) + \beta} - \frac{\beta e^{(\lambda(\alpha)+\beta)t_1}}{\lambda(\alpha) + \beta} + \beta t_1 - 1 \right) \tag{9}$$

Therefore, the order quantity per cycle can be computed as

$$Q = D_\tau + \int_0^{\frac{T}{n}} D(p,t)dt \tag{10}$$

The solution of Equation (10) is given by

$$Q = \frac{D(p)}{\lambda(\alpha) + \beta}(e^{(\lambda(\alpha)+\beta)t_1} - 1) + D(p)\left(\frac{T}{n} - t_1\right) \tag{11}$$

Here, the maximum number of backorder unit per cycle are as follows

$$S\left(\frac{T}{n}\right) = D(p)\left(\frac{T}{n} - t_1\right) \tag{12}$$

To find total profit function, first we calculate sales revenue and various types of costs.
Sales Revenue (SR) of the inventory model is

$$SR = np\int_0^{\frac{T}{n}} D(p,t)dt,$$

$$SR = \frac{np\beta D(p)}{(\lambda(\alpha) + \beta)^2}\left(e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha) + \beta)t_1 - 1\right) + pD(p)T \tag{13}$$

whereas the purchase Cost (PC) is

$$PC = ncQ$$

substituting the value of Q from Equation (11)

$$PC = \frac{ncD(p)}{\lambda(\alpha)+\beta}(e^{(\lambda(\alpha)+\beta)t_1} - 1) + ncD(p)\left(\frac{T}{n} - t_1\right) \tag{14}$$

The holding Cost (HC) of the complete inventory is

$$HC = \int_0^{t_1} nc_h I(t)dt,$$

$$\begin{aligned} HC = -\frac{na_0D(p)t_1}{\lambda(\alpha)+\beta} - \frac{na_0D(p)}{(\lambda(\alpha)+\beta)^2} + \frac{na_0D(p)}{(\lambda(\alpha)+\beta)^2}e^{(\lambda(\alpha)+\beta)t_1} - \frac{na_1D(p)t_1^2}{2(\lambda(\alpha)+\beta)} \\ -\frac{na_1D(p)t_1}{(\lambda(\alpha)+\beta)^2} - \frac{na_1D(p)}{(\lambda(\alpha)+\beta)^3} + \frac{na_1D(p)}{(\lambda(\alpha)+\beta)^3}e^{(\lambda(\alpha)+\beta)t_1} \end{aligned} \tag{15}$$

and shortage Cost (SC) of the inventory which is faced in the interval $\left[t_1, \frac{T}{n}\right]$ is

$$SC = ns\int_{t1}^{\frac{T}{n}} (-I(t))dt,$$

$$SC = nsD(p)\left(\frac{T^2}{2n^2} + \frac{t_1^2}{2} - \frac{Tt_1}{n}\right) \tag{16}$$

Ordering Cost (OC) and preservation technology Cost (PTC) of complete inventory model is

$$OC = nA \tag{17}$$

and

$$PTC = \alpha T, \tag{18}$$

Cost due to deterioration face by inventory model (DC) is

$$DC = ndD_\tau.$$

Putting the value of D_τ from Equation (9) in Equation (18) the value of (DC) is calculated as

$$DC = \frac{ndD(p)}{\lambda(\alpha)+\beta}\left(e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha)+\beta)t_1 + \frac{\beta}{\lambda(\alpha)+\beta} - \frac{\beta e^{(\lambda(\alpha)+\beta)t_1}}{\lambda(\alpha)+\beta} + \beta t_1 - 1\right) \tag{19}$$

Carbon Emission Cost (CEC) of the inventory is calculated by variable and fixed carbon emission factor c_v and c_f, m be the weight of the product and γ is the tariff on the carbon emission done by the model. CEC is given by

$$CEC = (c_v mQ + c_f)\gamma \tag{20}$$

$$CEC = \frac{c_v m\gamma D(p)}{\lambda(\alpha)+\beta}(e^{\lambda(\alpha)+\beta)t_1} - 1) + c_f\gamma \tag{21}$$

Total profit function (TP) is evaluated by subtracting all costs of inventory from the sales revenue

$$TP = SR - (PC + HC + SC + DC + OC + PTC + CEC). \tag{22}$$

Putting all the required values from Equations (13), (14), (15), (16), (17), (18), (19) and (121), the total profit function is computed as follows

$$\begin{aligned}TP = &\frac{np\beta D(p)}{\lambda(\alpha)+\beta}\left(e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha)+\beta)t_1 - 1\right) + pD(p)T - \frac{ncD(p)}{\lambda(\alpha)+\beta}(e^{(\lambda(\alpha)+\beta)t_1} - 1)\\ &-ncD(p)\left(\frac{T}{n} - t_1\right) + \frac{na_0D(p)t_1}{\lambda(\alpha)+\beta} + \frac{na_0D(p)}{(\lambda(\alpha)+\beta)^2} - \frac{na_0D(p)}{(\lambda(\alpha)+\beta)^2}e^{(\lambda(\alpha)+\beta)t_1} + \frac{na_1D(p)t_1^2}{2(\lambda(\alpha)+\beta)}\\ &+\frac{na_1D(p)t_1}{(\lambda(\alpha)+\beta)^2} + \frac{na_1D(p)}{(\lambda(\alpha)+\beta)^3} - \frac{na_1D(p)}{(\lambda(\alpha)+\beta)^3}e^{(\lambda(\alpha)+\beta)t_1} - nsD(p)\left(\frac{T^2}{2n^2} + \frac{t_1^2}{2} - \frac{Tt_1}{n}\right)\\ &-\frac{ndD(p)}{\lambda(\alpha)+\beta}\left(e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha)+\beta)t_1 + \frac{\beta}{\lambda(\alpha)+\beta} - \frac{\beta e^{(\lambda(\alpha)+\beta)t_1}}{\lambda(\alpha)+\beta} + \beta t_1 - 1\right)\\ &-nA - \alpha T - \frac{c_v m\gamma D(p)}{\lambda(\alpha)+\beta}(e^{\lambda(\alpha)+\beta)t_1} - 1) - c_f\gamma\end{aligned} \tag{23}$$

This is a non-linear optimization problem with single objective. To find the optimal values of t_1 and T, the partial derivatives of objective function TP with respect to t_1 and T are solved by putting them equal to 0. The following equations are solved to find the optimal values.

$$\frac{\partial(TP)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial(TP)}{\partial T} = 0 \tag{24}$$

$$\begin{aligned}\frac{\partial(TP)}{\partial t_1} = &\frac{np\beta D(p)}{\lambda(\alpha)+\beta}e^{(\lambda(\alpha)+\beta)t_1} - \frac{np\beta D(p)}{\lambda(\alpha)+\beta} - ncD(p)e^{(\lambda(\alpha)+\beta)t_1} + ncD(p) + \frac{a_0D(p)}{\lambda(\alpha)+\beta}\\ &-\frac{a_0D(p)}{\lambda(\alpha)+\beta}e^{(\lambda(\alpha)+\beta)t_1} + \frac{a_1D(p)t_1}{\lambda(\alpha)+\beta} + \frac{a_1D(p)}{(\lambda(\alpha)+\beta)^2} - \frac{a_1D(p)}{(\lambda(\alpha)+\beta)^2}e^{(\lambda(\alpha)+\beta)t_1}\\ &-nsD(P)t_1 + sD(p)T - ndD(p)(\lambda(\alpha)+\beta)e^{(\lambda(\alpha)+\beta)t_1} + ndD(p)(\lambda(\alpha)+\beta)\\ &+ndD(p)\beta e^{(\lambda(\alpha)+\beta)t_1} - c_v m\gamma D(p)e^{(\lambda(\alpha)+\beta)t_1} = 0\end{aligned} \tag{25}$$

and

$$\frac{\partial(TP)}{\partial T} = pD(p) - cD(p) - \frac{sD(p)T}{n} + sD(p)t_1 - \alpha = 0 \tag{26}$$

The optimal values t_1^* and T^* can be calculated by solving Equation (25) and (26). Maximum value of total profit function (TP^*) is calculated by using the newly found values of t_1^* and T^*. Furthermore, the necessary conditions for the maximization of objective function are

$$\left(\frac{\partial^2TP}{\partial t_1^2}\right)\left(\frac{\partial^2TP}{\partial T^2}\right) - \left(\frac{\partial^2TP}{\partial t_1\partial T}\right)^2 > 0 \;\; and \;\; \left(\frac{\partial^2TP}{\partial t_1^2}\right)^2 < 0, \; \left(\frac{\partial^2TP}{\partial T^2}\right)^2 < 0. \tag{27}$$

The partial derivatives to prove the necessary conditions are

$$\frac{\partial^2(TP)}{\partial t_1^2} = np\beta D(p)\, e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha)+\beta)\, ncD(p)\, e^{(\lambda(\alpha)+\beta)t_1} - a_0 D(p)\, e^{(\lambda(\alpha)+\beta)t_1} + \frac{a_1 D(p)}{\lambda(\alpha)+\beta} - \frac{a_1 D(p)}{\lambda(\alpha)+\beta} e^{(\lambda(\alpha)+\beta)t_1} - nsD(P) - (\lambda(\alpha)+\beta)^2\, ndD(p)\, e^{(\lambda(\alpha)+\beta)t_1} + (\lambda(\alpha)+\beta)\, ndD(p)\, \beta e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha)+\beta)\, c_v m\gamma D(p)\, e^{(\lambda(\alpha)+\beta)t_1}, \tag{28}$$

$$\frac{\partial^2(TP)}{\partial T^2} = -\frac{sD(p)}{n}, \tag{29}$$

and

$$\frac{\partial^2(TP)}{\partial t_1 \partial T} = sD(p) \tag{30}$$

The concavity of the TP is shown numerically by satisfying the necessary conditions in Equation (27) in the next Section. The optimal values t_1^* and T^* is obtained using software Mathematica 11.

An algorithm to find the total profit function is shown in steps as follows:

Step 1: Input: Write all the parameters values in Equations (25) and (26).

Step 2: Solve both Equations (25) and (26) using Mathematica 11 for optimal values t_1^* and T^*.

Step 3: Verify the concavity of the profit function by plotting a 3D graph.

Step 4: Find the maximum total profit function TP^* using values t_1^* and T^*.

Step 5: Output: The output is the total profit function.

A flowchart of the proposed solution procedure is presented in the following Figure 3.

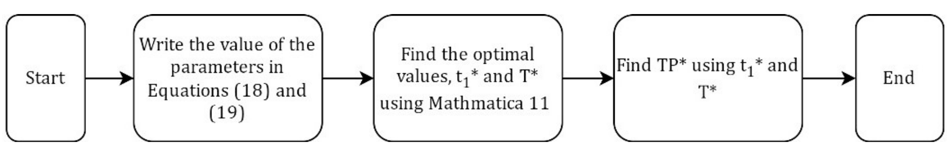


Figure 3: *Flowchart of the proposed solution procedure.*

5. Numerical experimentation

To illustrate the proposed mathematical model, a numerical example is taken and a 3D graph is modeled to show the concavity of the profit function along with the necessary conditions. Sensitivity analysis is also done for the numerical example.

Example: The parameter values in proper units are considered for this example are $n = 4$, $D(p) = \frac{a}{b^p}$ where $a = 50$, $b = 0.015$, $p = \$500$ /unit, $\alpha = \$2$ /unit/unit time , $\beta = 0.01$, $a_0 = 110$, $a_1 = 3$, $s = \$12$/unit , $d = \$0.5$/unit , $A = \$500$, $c = \$2$/unit , $\delta = 0.4$ and $\lambda_0 = 0.01$. Applying the above method the optimal solution for total profit.

Using the solution procedure, the optimal values are $t^* = 22.536$ and $T^* = 256$ whereas, total profit is \$1125440.

Based on the above data, the concavity is demonstrated numerically by using the conditions in Equation (27). The sufficient conditions are

$$\frac{\partial^2(TP)}{\partial t_1^2} = -5031.8521 < 0$$

$$\frac{\partial^2(TP)}{\partial T^2} = -60 < 0$$

$$\left(\frac{\partial^2 TP}{\partial t_1^2}\right)\left(\frac{\partial^2 TP}{\partial T^2}\right) - \left(\frac{\partial^2 TP}{\partial t_1 \partial T}\right)^2 = 302871.174 > 0$$

The variations of total profit function with respect to t_1 and T is shown graphically in the following Figure 4.

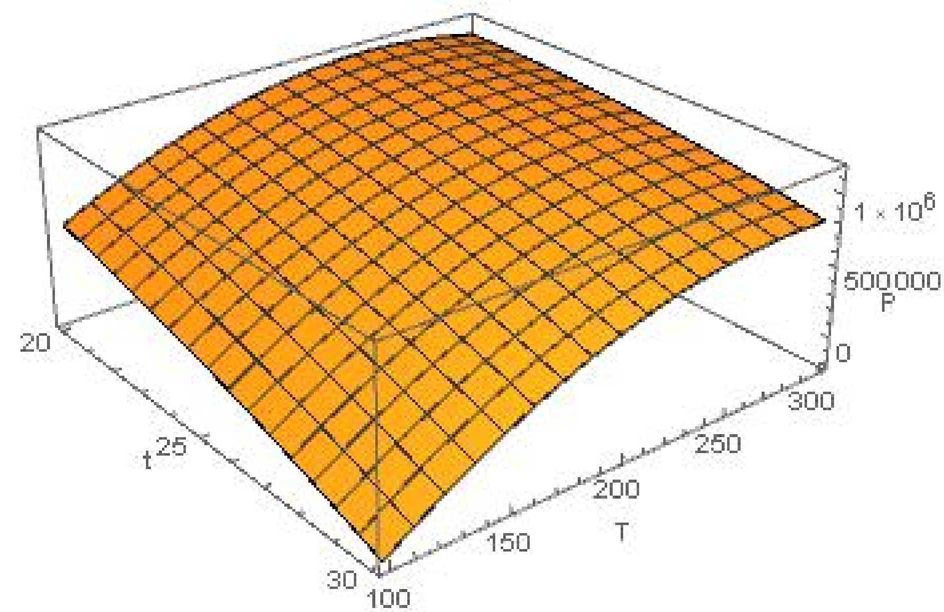


Figure 4: *Total profit function in relation with time t_1 and cycle length T.*

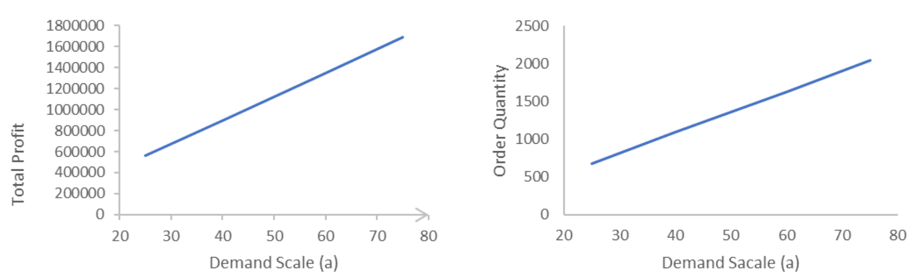
6. Sensitivity analysis

In order to demonstrate the effect of various parameters with respect to total profit and order quantity, the sensitivity analysis is carried out. In this analysis, at a time the value of one parameter is changed while remaining parameters are kept unchanged. All of this is summarized in the Table 3.

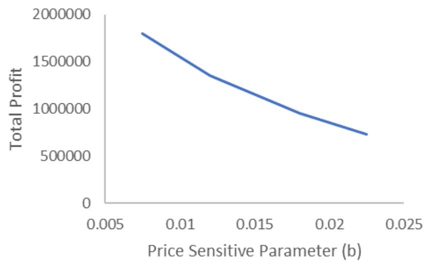
Parameters	**% Change**	**Total Profit** (TP)	**% Change in** (TP)	**Order Quantity** (Q)	**% Change in** (Q)
a	-50%	561460	-50.11	680	-50
	-20%	899487	-20.04	1088	-20
	20%	1351030	20.04	1632	20
	50%	1689420	50.11	2041	50.07
b	-50%	1802210	60.13	2177	60.07
	-20%	1351030	20.04	1632	20
	20%	956245	-15.03	1156	-15
	50%	730654	-35.08	884	-35
λ_0	-50%	1091720	-3	1348	-0.88
	-20%	1089450	-3.2	1356	-0.29
	20%	1081060	-3.94	1368	0.59
	50%	1074920	-4.49	1376	1.18
δ	-50%	1074490	-4.53	1376	1.18
	-20%	1081610	-3.89	1367	0.51
	20%	1088710	-3.26	1359	-0.15
	50%	1092680	-2.91	1353	-0.51
β	-50%	1109450	-1.42	1331	-2.13
	-20%	1095170	-2.69	1349	-0.81
	20%	1075530	-4.43	1375	1.1
	50%	1060270	-5.79	1395	2.57
α	-50%	1074740	-4.5	1376	1.18
	-20%	1081660	-3.89	1367	0.51
	20%	1088600	-3.27	1358	-0.15
	50%	1092420	-2.93	1353	-0.51
n	-50%	-769195	-131.65	2640	94.12
	-20%	505828	-55.06	1787	31.4
	20%	1482930	31.76	1104	-18.82
	50%	1709430	51.89	934	-31.32

Table 3: *Sensitivity analysis of numerical example.*

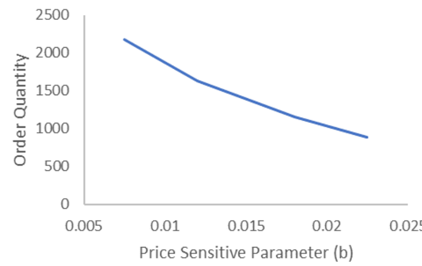
The variations in total profit and order quantity in relation with different parameters are represented graphically in Figure 5.



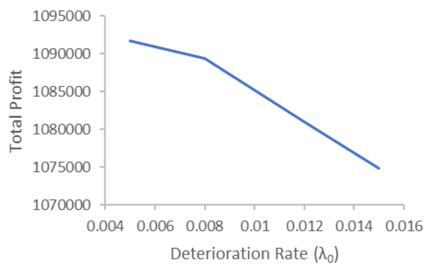
5(a) 5(b)



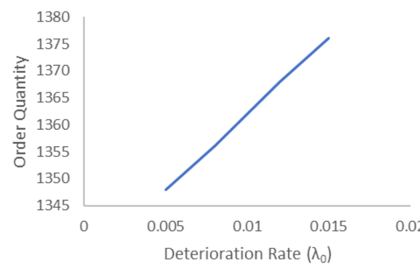
5(c)



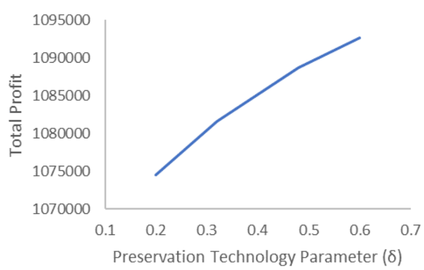
5(d)



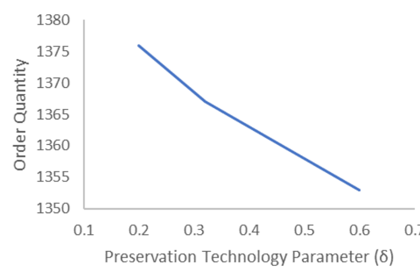
5(e)



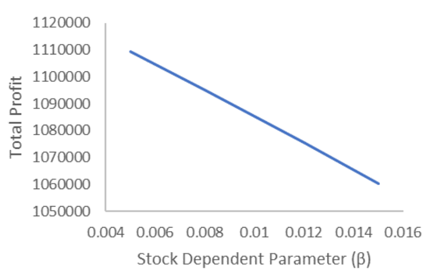
5(f)



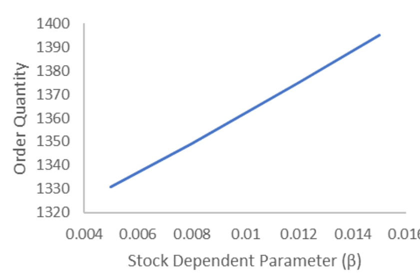
5(g)



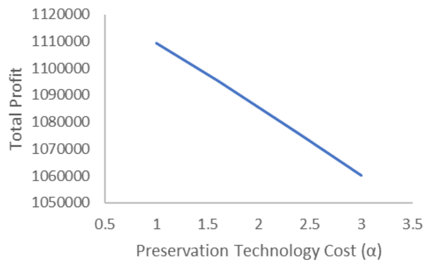
5(h)



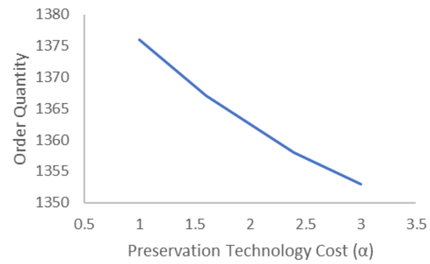
5(i)



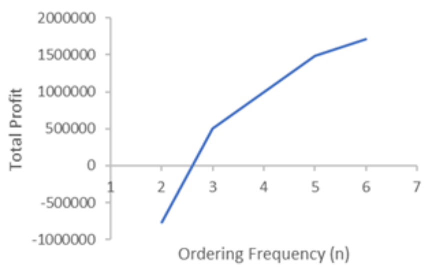
5(j)



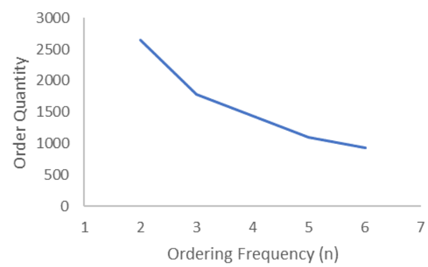
5(k)



5(l)



5(m)



5(n)

Figure 5: *Variations in total profit and order quantity.*

7. Results and managerial insights

Based on numerical example and sensitivity analysis, the following points are observed

a) Total Profit directly varies with a, δ, n as seen in Figures 5(a, g, m) respectively and it is inversely varying with the values of $b, \alpha, c, \lambda_0, \beta$ as noticed in Figures 5(c, e, i, k).

b) There is a sudden change in total profit with respect to ordering frequency n as shown in Figure 5(m)

c) Order Quantity directly varies with a, λ_0, β as seen in Figures 5(b, f ,j) and it is inversely varying with b, δ, α, n noticed in Figures 5(d, h, l, n).

d) Cost price c does not have any effect on order quantity.

e) PRT cost has very minimal effect on order quantity as seen in Figure 5(n).

The managerial insights of the inventory model are observed such that

- An increase in demand scale parameter a results in the increment of total profit as well as order quantity. Retailers should try to increase their demand for more profit.
- Increase in preservation technology parameter δ means high profit and less order quantity. Decision makers should increase their investment in preservation technology to get more profit with less order quantity.

8. Conclusions

In this paper, a profit optimization model was developed for sustainable inventory with demand rate dependent on price-stock considering the preservation technology. In addition, the shortages were allowed which was completely backlogged as they are very practical in nature. The total profit function was developed with sales revenue and various other types of costs. The main findings of this paper were summarized in terms of optimal values for the total profit and order quantity. In addition, it was observed that total profit varies directly with demand scale and price sensitive parameter. Whereas, increase in number of cycles result in increase of total profit and decrease in order quantity. This paper can be expanded in multiple directions. For example, one can consider the time dependent demand in place of price and stock dependent demand. Another possible extension is to consider the uncertainty in modal parameters, for instance fuzzy number, interval number, type-1 fuzzy sets, intuitionistic fuzzy sets, etc.

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